Solutions to Homework Assignment 16

MATH 249

Section 14.5, Page 907 Stewart 6e

1, 4, 5, 8, 10, 11, 15, 23, 25, 31, 33, 35, 40, 41, 43

5. $w = xe^{y/z}, x = t^2, y = 1-t, z = 1+2t$. $\dfrac{dw}{dt} = e^{y/z}(2t) + \dfrac{x}{z}e^{y/z}(-1) - \dfrac{xy}{z^2}e^{y/z}(2) = e^{y/z}\left(2t - \dfrac{x}{z} - \dfrac{2xy}{z^2}\right)$.

15. $g_u(0,0) = f_x(e^0 + \sin 0, e^0 + \cos 0)\left.\dfrac{\partial x}{\partial u}\right|_{(0,0)} + f_y(e^0 + \sin 0, e^0 + \cos 0)\left.\dfrac{\partial y}{\partial u}\right|_{(0,0)} = 2(e^0) + 5(e^0) = 7$. $g_v(0,0)$ is similar.

23. $\dfrac{\partial R}{\partial x} = \dfrac{2u}{u^2+v^2+w^2}(1) + \dfrac{2v}{u^2+v^2+w^2}(2) + \dfrac{2w}{u^2+v^2+w^2}(2y)$. At $(1,1)$, $u = 3, v = 1, w = 2$, so we get $\dfrac{1}{14}(6+4+8) = \dfrac{9}{7}$. The other part is similar.

31. Let $F(x,y,z) = x^2 + y^2 + z^2 - 3xyz$. If we set $F(x,y,z) = 0$, we find that $\dfrac{\partial}{z}\partial x = -\dfrac{2x - 3yz}{2z - 3xy}$ and $\dfrac{\partial}{z}\partial y = -\dfrac{2y - 3xz}{2z - 3xy}$.

35. $\dfrac{dT}{dt} = \dfrac{\partial T}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial T}{\partial y}\dfrac{dy}{dt} = T_x\dfrac{1}{2\sqrt{1+t}} + T_y \cdot \dfrac{1}{3}$. At $t = 3$, $(x,y) = (2,3)$, so we have $T'(3) = 4\dfrac{1}{4} + 3 \cdot \dfrac{1}{3} = 2$ degrees Celsius per second. Pretty fast!

41. $V = \dfrac{8.31T}{P}$ according to Example 2. $\dfrac{dV}{dt} = \dfrac{8.31}{P}T'(t) - \dfrac{8.31T}{P^2}P'(t)$. T' and P' are given to us as constants, so we can compute $V' = 8.31(0.15/20 - 320(0.05)/20^2) \approx -0.27$ liters per second.

43. $\dfrac{\partial z}{\partial r} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial r} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial r} = z_x \cos\theta + z_y \sin\theta$. Also, $\dfrac{\partial z}{\partial \theta} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial \theta} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial \theta} = -z_x r\sin\theta + z_y r\cos\theta$. It is not hard to see now that $\left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2 = \left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2$.