Solutions to Homework Assignment 17

MATH 249

Section 14.6, Page 920 Stewart 6e

1, 4, 5, 7, 9, 10, 12, 13, 17, 23, 26, 31, 36, 39, 42, 47

9. (a) $\nabla f(x,y,z) =< e^{2yz}, 2xze^{2yz}, 2xye^{2yz} >$.

   (b) $\nabla f(3,0,2 >=< 1,12,0 >$.

   (c) $D_u f(3,0,2) =< 1,12,0 > \cdot < 2/3, -2/3, 1/3 >= \dfrac{-22}{3}$.

12. $\nabla f(x,y) = \left\langle \dfrac{2x}{x^2+y^2}, \dfrac{2y}{x^2+y^2} \right\rangle$, so $\nabla f(2,1) =< 4/5, 2/5 >$. Now $D_u f(2,1) =< 4/5, 2/5 > \cdot \dfrac{1}{\sqrt{5}} < -1, 2 >= 0$.

17. $\nabla g(x,y,z) = \dfrac{3}{2}(x+2y+3z)^{1/2} < 1,2,3 >$, so $\nabla g(1,1,2) = \dfrac{3}{2}(3) < 1,2,3 >= \dfrac{9}{2} < 1,2,3 >$. Thus $D_u f(1,1,2) = \dfrac{9}{2} < 1,2,3 > \cdot \dfrac{1}{\sqrt{5}} < 0,2,-1 >= \dfrac{9}{2\sqrt{5}}$.

23. $\nabla f(x,y) =< y\cos xy, x\cos xy >$, so $\nabla f(1,0) =< 0,1 >$. The direction of greatest change is in the direction of the y-axis and its magnitude is 1.

31. We are given that $T(x,y,z) = \dfrac{k}{\sqrt{x^2+y^2+z^2}}$. Since $T(1,2,2) = 120°$, $120 = \dfrac{k}{3}$ and $k = 360$.

   (a) $\nabla T(x,y,z) = \dfrac{-360}{(x^2+y^2+z^2)^{3/2}} < x,y,z >$, so $\nabla T(1,2,2) = -\dfrac{40}{3} < 1,2,2 >$. The direction we wish to travel is $< 1,-1,1 >$, so $u = \dfrac{1}{\sqrt{3}} < 1,-1,1 >$. We get $D_u f(1,2,2) = -\dfrac{40}{3} < 1,2,2 > \cdot \dfrac{1}{\sqrt{3}} < 1,-1,1 >= -\dfrac{40}{3\sqrt{3}}$.

   (b) Since $\nabla T(x,y,z) = \dfrac{-360}{(x^2+y^2+z^2)^{3/2}} < x,y,z >$, we have a gradient in the direction of $- < x,y,z >$ at the point (x,y,z), so this points toward the origin.