Solutions to Homework Assignment 20

MATH 249

Section 15.2, Page 964 Stewart 6e

3, 6, 7, 8, 10, 12, 16, 17, 18, 20, 21, 26, 29

3. $\int_1^3 \int_0^1 (1+4xy)dxdy = \int_1^3 x + 2x^2y\Big|_{x=0}^1 dy = \int_1^3 1+2y\ dy = y + y^2\Big|_1^3 = (3+3^2) - (1+1^2) = 10.$

7. $\int_0^2 \int_0^1 (2x+y)^8 dxdy = \int_0^2 \frac{1}{18}(2x+y)^9\Big|_0^1 dy = \int_0^2 \frac{1}{18}[(y+2)^9 - y^9]\ dy = \frac{1}{180}[(y+2)^{10} - y^{10}]\Big|_0^2 = \frac{1}{180}(4^{10} - 2^{10} - 2^{10}) = \frac{1046528}{180}.$

12. The inner integral is $\int_0^1 xy(x^2+y^2)^{1/2}\ dy = \frac{1}{3}x(x^2+y^2)^{3/2}\Big|_0^1 = \frac{1}{3}x(x^2+1)^{3/2} - \frac{1}{3}x(x^2)^{3/2}$. The outer integral becomes $\int_0^1 \frac{1}{3}x(x^2+1)^{3/2} - \frac{1}{3}x^4\ dx = \frac{1}{15}(x^2+1)^{5/2} - \frac{1}{15}x^5\Big|_0^1 = \frac{2^{5/2}-1}{15} - \frac{1}{15} = \frac{4\sqrt{2}-2}{15}.$

16. This double integral is equal to the iterated integral $\int_0^{\pi/2}\int_0^{\pi} \cos(x+2y)dxdy$. The inner integral is $\int_0^{\pi} \cos(x+2y)\ dx = \sin(x+2y)\Big|_0^{\pi} = \sin(2y+\pi) - \sin 2y = -2\sin 2y$ since $\sin(\theta+\pi) = -\sin\theta$. The outer integral becomes $-2\int_0^{\pi/2} \sin 2y\ dy = \cos 2y\Big|_0^{\pi/2} = -1-1 = -2.$

17. This double integral is equal to the iterated integral $\int_0^1\int_{-3}^3 \frac{xy^2}{x^2+1}dydx$. The inner integral is $\int_{-3}^3 \frac{xy^2}{x^2+1}\ dy = \frac{xy^3}{3(x^2+1)}\Big|_{-3}^3 = \frac{18x}{x^2+1}$. The outer integral is then $\int_0^1 \frac{18x}{x^2+1}\ dx = 9\ln(x^2+1)\Big|_0^1 = 9\ln 2.$

20. This double integral is equal to the iterated integral $\int_0^1\int_0^1 \frac{x}{1+xy}dydx$. For the inner integral, use $u = 1+xy$. Then $du = xdy$, so we get $\int_1^{x+1} \frac{1}{u}\ du = \ln u\Big|_1^{x+1} = \ln(x+1)$. The outer integral is then $\int_0^1 \ln(x+1)\ dx = (x+1)\ln(x+1) - (x+1)\Big|_0^1 = 2\ln 2 - 2 + 1 = 2\ln 2 - 1$. (To find $\int \ln w = w\ln w - w$, integrate by parts.)

21. This double integral is equal to the iterated integral $\int_0^2\int_0^1 xye^{x^2y}dxdy$. The inner integral is $\int_0^1 xye^{x^2y}\ dx = \frac{1}{2}e^{x^2y}\Big|_0^1 = \frac{1}{2}(e^y - 1)$. The outer integral is then $\int_0^2 \frac{1}{2}(e^y-1)\ dy = \frac{1}{2}(e^y - y)\Big|_0^2 = \frac{1}{2}(e^2-2-1) = \frac{1}{2}(e^2-3).$