

# Solutions to Homework Assignment 21

MATH 249

Section 15.3 Stewart 6e, Page 972

4, 6, 7, 10, 11, 15, 18, 19, 22, 23, 24, 25, 28, 39, 41, 44, 46, 47, 50

$$4. \int_1^2 \int_y^2 xy dx dy = \int_1^2 \frac{1}{2} x^2 y \Big|_y^2 dy = \int_1^2 2 dy = - \Big|_1^2 \frac{y^3}{2} yy^2 - \frac{1}{8} y^4 = \frac{9}{8}.$$

$$5. \int_0^{\pi/2} \int_0^{\cos \theta} e^{\sin \theta} r dr d\theta = \int_0^{\pi/2} r e^{\sin \theta} \Big|_0^{\cos \theta} d\theta = \int_0^{\pi/2} \cos \theta e^{\sin \theta} d\theta = e^{\sin \theta} \Big|_0^{\pi/2} = e - 1.$$

15. This will be easier if we integrate with respect to  $x$  first. The  $y$ -values range from 1 up to 2, while the  $x$ -values range from the left line, given by  $x = 2 - y$ , to the right line, given by  $x = 2y - 1$ . We get

$$\int_1^2 \int_{2-y}^{2y-1} y^3 dx dy = \int_1^2 y^3 x \Big|_{x=2-y}^{x=2y-1} dy = \int_1^2 y^3 (2y-1-2+y) dy = \int_1^2 (3y^4 - 3y^3) dy = \frac{3}{5} y^5 - \frac{3}{4} y^4 \Big|_1^2 = \frac{93}{5} - \frac{45}{4} = \frac{147}{20}.$$

19. We have  $z = x + 2y$ . The boundary curves meet at  $x = 0$  and  $x = 1$  and  $x^4 \leq x$  on this interval. The volume is thus

$$\int_0^1 \int_{x^4}^x x + 2y dy dx = \int_0^1 xy + y^2 \Big|_{y=x^4}^{y=x} dx = \int_0^1 (x^2 + x^2) - (x^5 + x^8) dx = \int_0^1 2x^2 - x^5 - x^8 dx = \frac{2}{3} x^3 - \frac{1}{6} x^6 - \frac{1}{9} x^9 \Big|_0^1 = \frac{7}{18}.$$

22. On the triangle described,  $x$  goes from 0 to 1 and  $y$  goes from  $x$  to 1. We have

$$\int_0^1 \int_x^1 x^2 + 3y^2 dy dx = \int_0^1 x^2 y + y^3 \Big|_{y=x}^{y=1} dx = \int_0^1 (x^2 + 1) - (x^3 + x^3) dx = \int_0^1 x^2 + 1 - 2x^3 dx = \frac{1}{3} x^3 + x - \frac{1}{2} x^4 \Big|_0^1 = \frac{5}{6}.$$

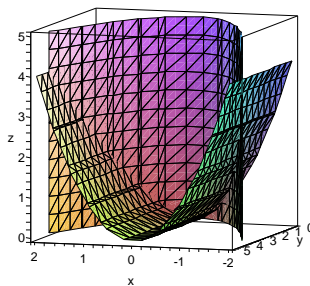
24. The lines  $y = x$  and  $x + y = 2$  in the  $xy$ -plane meet at  $(1, 1)$ . The plane  $z = x$  meets the plane  $z = 0$  on the  $y$ -axis. This means that the bounded region in the plane is the triangle with vertices  $(0, 0)$ ,  $(1, 1)$ , and  $(0, 2)$ . This is easier if we integrate with respect to  $y$  first:

$$\int_0^1 \int_x^{2-x} x dx dy = \int_0^1 xy \Big|_{y=x}^{y=2-x} = \int_0^1 [x(2-x) - x^2] dx = \int_0^1 2x - 2x^2 dx = x^2 - \frac{2}{3} x^3 \Big|_0^1 = \frac{1}{3}.$$

25.  $y$  is constrained between  $x^2$  and 4. This means that  $-2 \leq x \leq 2$ . We have

$$\int_{-2}^2 \int_{x^2}^4 x^2 dy dx = \int_{-2}^2 x^2 y \Big|_{y=x^2}^{y=4} = \int_{-2}^2 4x^2 - x^4 dx = \frac{4}{3} x^3 - \frac{1}{5} x^5 \Big|_{-2}^2 = \frac{128}{15}.$$

The region is graphed below; we have found the volume under the upward-opening parabolic cylinder.



28. We have  $z = \pm \sqrt{r^2 - y^2}$ . Because of the symmetry in this problem, I will just use  $z = \sqrt{r^2 - y^2}$

and double the result. In fact, the boundary in the  $xy$ -plane is  $x = \pm\sqrt{r^2 - y^2}$ , so again I can use symmetry to double the result obtained by letting  $x$  range from 0 to  $\sqrt{r^2 - y^2}$ . Here, I am anticipating having to integrate  $\sqrt{r^2 - y^2}$ , which is hard if I integrate with respect to  $y$ . For that reason, I will integrate with respect to  $x$  first.

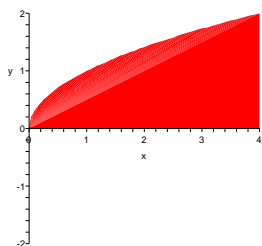
Finally, inside this region  $y$  ranges from  $-r$  to  $r$ . Symmetry once more allows me to integrate from 0 to  $r$  and double the result.

$$\text{We get } V = 8 \int_0^r \int_0^{\sqrt{r^2 - y^2}} \sqrt{r^2 - y^2} dx dy = 8 \int_0^r \sqrt{r^2 - y^2} x \Big|_{x=0}^{x=\sqrt{r^2 - y^2}} dy = 8 \int_0^r r^2 - y^2 dy = 8r^2y - \frac{8}{3}y^3 \Big|_0^r = \frac{16}{3}r^3.$$

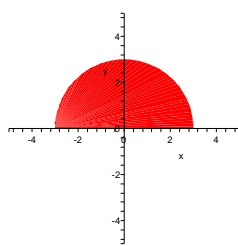
39. The red shaded region is the region of integration. Switching gives  $\int_0^2 \int_{y^2}^4 f(x, y) dx dy$ .

41. The red shaded region is the region of integration. Switching gives  $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} dy dx$ .

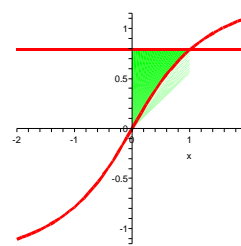
44. The green shaded region is the region of integration. On the boundary  $y = \arctan x$ , we have  $x = \tan y$ .  $y = \pi/4$  meets  $y = \arctan x$  at  $x = 1$ . Switching the order gives  $\int_0^{\pi/4} \int_0^{\tan y} f(x, y) dx dy$ .



Number 37



Number 39



Number 42

50. Switching the order of integration gives  $\int_0^2 \int_0^{x^3} e^{x^4} dy dx = \int_0^2 x^3 e^{x^4} dx = \frac{1}{4} e^{x^4} \Big|_0^2 = \frac{1}{4}(e^{16} - 1)$ .