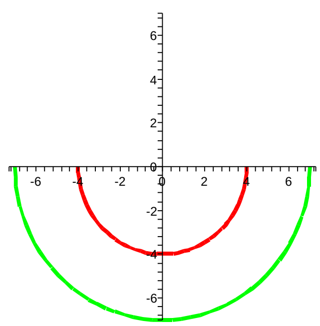
Solutions to Homework Assignment 22

MATH 249

Section 15.4 Stewart 6e, Page 978

1-5, 7, 10, 13, 16, 18, 19, 25, 26, 29, 30

1. I would use polar coordinates. $\int_0^{3\pi/2}\int_0^4 f(r\cos\theta, r\sin\theta)r\,dr\,d\theta$.
2. I would use rectangular coordinates. $\int_{-1}^{1}\int_0^{1-x^2} f(x,y)\,dy\,dx$.
3. I would use rectangular coordinates. $\int_{-1}^{1}\int_0^{(x+1)/2} f(x,y)\,dy\,dx$.
4. I would use polar coordinates. $\int_{-\pi/2}^{\pi/2}\int_3^6 f(r\cos\theta, r\sin\theta)r\,dr\,d\theta$.
5. $\int_\pi^{2\pi}\int_4^7 r\,dr\,d\theta = \int_\pi^{2\pi} \frac{1}{2}r^2\Big|_4^7 d\theta = \int_\pi^{2\pi}\frac{33}{2}\,d\theta = \frac{33}{2}\theta\Big|_\pi^{2\pi} = \frac{33\pi}{2}$. You can check that this is correct by finding the areas of the two semicircles and subtracting the smaller from the larger.



7. $\int_0^{2\pi}\int_0^3 r^2\sin\theta\cos\theta\, r\,dr\,d\theta = \int_0^{2\pi}\frac{1}{4}r^4\sin\theta\cos\theta\Big|_0^3 d\theta = \int_0^{2\pi}\frac{81}{4}\pi\sin\theta\cos\theta\;d\theta = \frac{81}{4}\pi\frac{1}{2}\sin^2\theta\Big|_0^{2\pi} = 0.$

13. The line $y = x$ is at an angle of $\theta = \frac{\pi}{4}$, so we have $\int_0^{\pi/4}\int_1^2 \arctan(\tan\theta)r\,dr\,d\theta = \int_0^{\pi/4}\frac{1}{2}r^2\theta\;d\theta = \frac{3}{4}\theta^2\Big|_0^{\pi/4} = \frac{3\pi^2}{64}$.

16. The graph is a cardioid. We have $\iint_R 1dA = \int_0^{2\pi}\int_0^{4+3\cos\theta} r\,dr\,d\theta = \int_0^{2\pi}\frac{1}{2}r^2\Big|_0^{4+3\cos\theta} d\theta = \int_0^{2\pi}\frac{1}{2}(16 + 24\cos\theta + 9\cos^2\theta)d\theta = \int_0^{2\pi} 8 + 12\cos\theta + \frac{9}{2}\frac{1+\cos 2\theta}{2}\;d\theta = 8\theta + 12\sin\theta + \frac{9}{4}(\theta + \sin(2\theta)/2)\Big|_0^{2\pi} = \frac{41}{2}\pi$.

21. $\int_0^{2\pi}\int_0^3 r^2 r\,dr\,d\theta = 2\pi\,\frac{1}{4}r^4\Big|_0^3 = \frac{81\pi}{2}$. Notice that since the inner integral is independent of θ we can write $\int_0^{2\pi}\int_0^3 r^2 r\,dr\,d\theta = \int_0^{2\pi}d\theta\int_0^3 r\,dr$ by factoring.

25. The sphere is $z = \sqrt{1-r^2}$ and the cone is $z = r$ since we want the part between the two. These meet above the circle $r = \frac{1}{\sqrt{2}}$. (Use $z^2 = x^2 + y^2$ from the cone and substitute into the sphere's equation.) We have $V = \int_0^{2\pi}\int_0^{1/\sqrt{2}}(\sqrt{1-r^2} - r)r\,dr\,d\theta = 2\pi\left(-\frac{1}{3}(1-r^2)^{3/2} - \frac{1}{3}r^3\Big|_0^{1/\sqrt{2}}\right) = 2\pi\left(-\frac{1}{3}(1/2)^{3/2} - \frac{1}{3}(1/2)^{3/2}) + \frac{1}{3}\right) =$

$\frac{\pi}{3}(2-\sqrt{2})$.

29. The upper limit on y is $\sqrt{9-x^2}$, so the boundary is some portion of $x^2+y^2=9$. Since x goes from -3 to 3 and y goes from 0 to the circle, we are integrating over the portion of the circle in the upper half-plane. We get $\int_0^{\pi}\int_0^3 \sin(r^2) r dr d\theta = \pi\left(-\frac{1}{2}\cos(r^2)\Big|_0^3\right) = \frac{\pi}{2}(1-\cos^2(9)) = \frac{\pi}{2}\sin^2(9)$.