Solutions to Homework Assignment 24

MATH 249

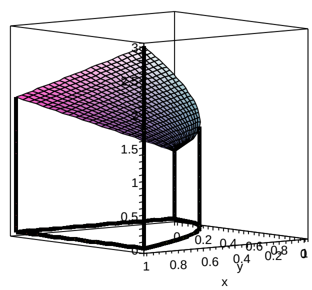
Section 15.7 Stewart 6e, Page 998

1, 4, 6, 11, 12, 13, 16, 19, 21, 23, 27, 28, 30

1. $\int_{-1}^{2}\int_{0}^{1}\int_{0}^{3} xyz^2 dzdxdy = \int_{-1}^{2}\int_{0}^{1} \frac{1}{3}xy(27)dxdy = \int_{-1}^{2} \frac{9}{2}y(1)dy = \frac{9}{4}(4-1) = \frac{27}{4}.$

6. $\int_{0}^{1}\int_{0}^{z}\int_{0}^{y} ze^{-y^2} dxdydz = \int_{0}^{1}\int_{0}^{z} yze^{-y^2} dydz = \int_{0}^{1} -\frac{1}{2}z(e^{-z^2}-1)dz = \frac{1}{4}e^{-z^2} + \frac{1}{4}z^2\Big|_{0}^{1} = \frac{1}{4}(1+e^{-1}-1) = f14e.$

11. The region in the xy-plane is below the graph of $y = \sqrt{x}$, above the x-axis, and to the left of $x = -1$. The region of integration is graphed below. We have $\int_{0}^{1}\int_{0}^{\sqrt{x}}\int_{0}^{1+x+y} 6xydzdydx = \int_{0}^{1}\int_{0}^{\sqrt{x}} 6xy(1+x+y)dydx = \int_{0}^{1} 3x(x) + 3x^2(x) + 2x(x^{3/2})dx = 1 + \frac{3}{4} + \frac{4}{7} = \frac{65}{28}.$



12. The region is the tetrahedron in the first octant with vertices $(0,0,0), (2,0,0), (0,2,0)$, and $(0,0,4)$. In the xy-plane, the region is bounded by the line $y = 2-x$. (Set $z = 0$ in the plane bounding the region above.) We have $\int_{0}^{2}\int_{0}^{2-x}\int_{0}^{4-2x-2y} ydzdydx = \int_{0}^{2}\int_{0}^{2-x} y(4-2x-2y)dydx = \int_{0}^{2} 2(2-x)^2 - x(2-x)^2 - \frac{2}{3}(2-x)^3dx =$
$\int_{0}^{2} 2(2-x)^2 - (4x-4x^2+x^3) - \frac{2}{3}(2-x)^3 \; dx = -\frac{2}{3}(2-x)^3 - 2x^2 + \frac{4}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{6}(2-x)^4\Big|_{0}^{2} =$
$\left(-8+\frac{32}{3}-4\right) - \left(-\frac{16}{3}+\frac{8}{3}\right) = \frac{4}{3}.$

13. You don't really need to see me integrate polynomials, so I will just set the rest of these up. The cylinder is an upside-down parabolic cylinder. It meets the plane $z = 0$ at $y = \pm 1$, so that gives the range for y. We get $\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{1-y^2} x^2e^y dzdydx = \frac{8}{3e}.$

19. The boundary in the xy-plane is $y = 4-2x$, so x ranges from 0 to 2 and the volume is $\int_{0}^{2}\int_{0}^{4-2x}\int_{0}^{4-2x-y} dzdydx = \frac{16}{3}$. We could also compute this using the formula for the volume of a pyramid: $V = \frac{1}{3}[(2)(4)/2](4) = \frac{16}{3}$. ($V = \frac{1}{3}Bh$.)

21. This is a natural candidate for polar coordinates. We have $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{1}^{5-y} dzdxdy =$
$\int_{0}^{2\pi}\int_{0}^{3}\left(\int_{1}^{5-r\sin\theta} dz\right) rdrd\theta = \int_{0}^{2\pi}\int_{0}^{3}(5r-5r^2\sin\theta) - rdrd\theta = \int_{0}^{2\pi} 2(9) - \frac{5}{3}(27)\sin\theta d\theta = 36\pi.$

23. (a) $\int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dzdydx$. This can be evaluated with a trigonometric substitution, but I will follow the book's advice and ask MAPLE.

(b) MAPLE says... $\frac{1}{4}\pi - \frac{1}{3}$.

27. While x ranges from 0 to 1, z ranges from 0 to the plane $1 - x$, which slants down over the xy-plane to meet it in the line $x = 1$. y ranges from 0 to $2 - 2z$; the latter gives a plane slanting down and meeting the xy-plane in the line $y = 2$. The result is a square-based pyramid with vertices $(0, 0, 0), (1, 0, 0), (0, 2, 0), (1, 2, 0)$, and $(0, 0, 2)$. See the back of the book for a graph.