Solutions to Homework Assignment 25

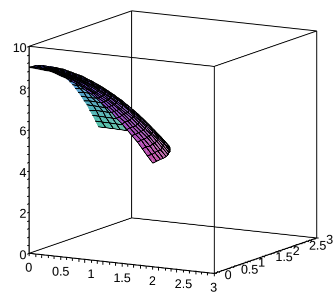
MATH 249

Section 15.7 Stewart 6e, Page 1004

1, 3, 5, 6, 7, 9, 15, 16, 17, 18, 20, 26, 29

15. The lower limit on z, $z = r$, gives a cone, while the upper limit is a horizontal plane. θ goes from 0 to 2π, so we take the cone all the way around the z-axis. Since r goes from 0 to 4, we get the entire (solid) cone up to $z = 4$. See the back of the book for a graph.

16. z ranges from the xy-plane to the upside-down paraboloid $9 - x^2 - y^2$, which meets the xy-plane in a circle of radius 3 centered at the origin. r only goes out to 2, so we have a cylinder of radius 2 cut off by the parabolic cap. Finally, since θ only ranges from 0 to $\pi/2$, we only get the portion of this in the first octant.



17. $\int_0^{2\pi}\int_0^4\int_{-5}^4 (r) r\,dz\,dr\,d\theta = (2\pi)(4^3/3)(9) = 384\pi.$

18. The paraboloid meets the xy-plane in the circle $x^2 + y^2 = 1$, so we get $\int_0^{\pi/2}\int_0^1\int_0^{1-r^2} (r^3\cos^3\theta + r^3\cos\theta\sin^2\theta) r\,dz\,dr\,d\theta = \int_0^{\pi/2}\int_0^1\int_0^{1-r^2} r^4\cos\theta\,dz\,dr\,d\theta = \frac{2}{35}.$

20. z ranges from 0 to $x + y + 5$, r ranges from 2 to 3, and θ ranges from 0 to 2π. (The plane that serves as a "lid" is entirely above the xy-plane over this circle, so we don't need to cut θ down any.) We get $\int_0^{2\pi}\int_2^3\int_0^{r\cos\theta + r\sin\theta + 5} (r\cos\theta) r\,dz\,dr\,d\theta = \frac{65\pi}{4}.$

26. The density is given by kr for some constant k. We have $\int_0^{2\pi}\int_0^a\int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} (kr) r\,dz\,dr\,d\theta = \frac{k\pi^2 a^4}{4}.$

29. (a) $W = \iiint_E h(P)g(P)\,dV.$

    (b) The cone rises from 0 feet to 12,400 feet over a 62,000-foot span, for a slope of $\frac{1}{5}$ down the side of the cone. Since the cone has circular symmetry, polar coordinates are natural for this problem. We get $\int_0^{2\pi}\int_0^{62000}\int_0^{12400-r/5} 200 z r\,dz\,dr\,d\theta \approx 3.1 \times 10^{19}$ foot-pounds.