Solutions to Homework Assignment 25

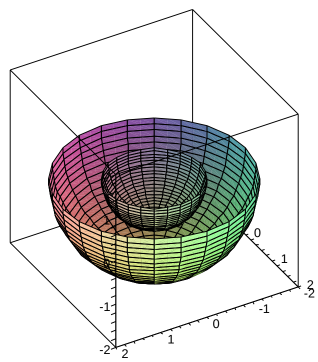
MATH 249-01 and -02

Section 15.8, Page 1037

1, 3, 5, 6, 7, 9, 11, 13, 17, 18, 20, 21, 23, 29, 35, 36

17. ρ ranges from 0 to 3, putting us inside a sphere of radius 3 centered at the origin. θ ranges from 0 to $\pi/2$, putting us above or below the first quadrant in the xy-plane. Finally, ϕ ranges from 0 to $\pi/6$, putting us inside a cone (and sphere) measuring 30 degrees down from the z-axis. Note that we only get the portion in the first octant. See the back of the book for a graph. (It's a quarter of a snow cone.)

18. Since ρ ranges from 1 to 2, we are inside a spherical shell with inner radius 1 and outer radius 2. Taking ϕ from $\pi/2$ to π puts us below the xy-plane, and taking θ from 0 to 2π gives us the entire shell below the xy-plane. We get $\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_1^2 \rho^2 \sin\phi d\rho d\phi d\theta = \frac{14}{3}\pi$.



20. In this one, ρ ranges from 1 to 2, θ from $\pi/2$ to 2π, and ϕ from 0 to $\pi/2$. We have
$\int_0^{\pi/2}\int_{\pi/2}^{2\pi}\int_1^2 f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi d\rho d\theta d\phi$.

21. $\int_0^{2\pi}\int_0^{\pi}\int_0^5 (\rho^4)\rho^2\sin\phi d\rho d\phi d\theta = \frac{4(5^7)}{7}\pi$.

23. $\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 (\rho\cos\phi)\rho^2\sin\phi d\rho d\phi d\theta = \frac{15}{16}\pi$.

29. The sphere in question sits on the origin and has a radius of 2: $\rho = 4\cos\phi$ implies that $\rho^2 = 4\rho\cos\phi$, or $x^2+y^2+z^2 = 4z$, so $x^2+y^2+(z-2)^2 = 4$. We get $\int_0^{2\pi}\int_0^{\pi/3}\int_0^{4\cos\phi} \rho^2\sin\phi d\rho d\phi d\theta = 10\pi$.

35. This looks spherical to me. The given cone can be written as $\phi = \pi/4$. The volume is
$\int_0^{2\pi}\int_0^{\pi/4}\int_0^1 \rho^2\sin\phi d\rho d\phi d\theta = \frac{2-\sqrt{2}}{3}\pi$.
By symmetry, the centroid will lie on the z-axis, so we only need to compute
$$\overline{z} = \frac{M_{xy}}{m} = \frac{\int_0^{2\pi}\int_0^{\pi/4}\int_0^1 (\rho\cos\phi)\rho^2\sin\phi d\rho d\phi d\theta}{\int_0^{2\pi}\int_0^{\pi/4}\int_0^1 \rho^2\sin\phi d\rho d\phi d\theta} = \frac{3}{8(\sqrt{2}-2)}.$$

36. Go spherical! $\int_0^{\pi/6}\int_0^{\pi}\int_0^a \rho^2\sin\phi d\rho d\phi d\theta = \frac{\pi a^3}{9}$.