Solutions to Homework Assignment 28

MATH 249

Section 16.2 Stewart 6e, Page 1043

1, 2, 4, 5, 6, 10, 13, 19-22, 34, 40, 42, 43

13. $dz = 2tdt$. We have $\int_0^1 (t^8)(2t)dt = \frac{1}{5}$.

21. $r'(t) =< 3t^2, -2t, 1 >$, so we have $\int_0^1 < \sin(t^3), \cos(-t^2), t^4 > \cdot < 3t^2, -2t, 1 >= \int_0^1 (3t^2 \sin t^3 - 2t \cos t^2 + t^4)dt = [-\cos(1) + 1] - \sin(1) + \frac{1}{5} = \frac{6}{5} - \sin 1 - \cos 1$.

22. $r'(t) =< 1, \cos t, -\sin t >$, so we get $\int_0^\pi < \cos t, \sin t, -t > \cdot < 1, \cos t, -\sin t > dt = \int_0^\pi (\cos t + \sin t \cos t + t \sin t)dt = \pi$.

34. (a) We simply need to add one for z; this would look like $\overline{z} = \frac{1}{m} \int_C z\rho(x, y, z)ds$. The other two just need $\rho(x, y)$ replaced with $\rho(x, y, z)$.

    (b) $ds = \sqrt{4\cos^2 t + 4\sin^2 t + 9}dt = \sqrt{13}dt$. Now $m = \int_0^{2\pi} k\sqrt{13}dt = 2\pi\sqrt{13}k$. Now $\overline{x}$ and $\overline{y}$ will both be 0 because of the symmetry of the helix.
    $\overline{z} = \frac{1}{2\pi\sqrt{13}k} \int_0^{2\pi} 3tk\sqrt{13}dt = \frac{1}{2\pi\sqrt{13}k} \frac{3k(4\pi^2\sqrt{13})}{2} = 3\pi$.

40. Let $x = t$ on $[-1, 2]$, so $y = t^2$ on $[-1, 2]$. The parabola is parameterized by $r(t) =< t, t^2 >$ on $[1, 3]$, so $r'(t) =< 1, 2t >$. The work is thus $\int_{-1}^2 < t \sin t^2, t^2 > \cdot < 1, 2t > dt = \int_{-1}^2 t \sin t^2 + 2t^3 dt = \frac{1}{2}(\cos 1 - \cos 4 + 15)$.

43. I will do this two ways. First, the hard way: the helix is parameterized by $r(t) =< 20\cos(6\pi t), 20\sin(6\pi t), 90t >$ on $[0, 1]$. I chose this parametrization in order to get a 20-foot radius, 3 complete revolutions on $[0, 1]$, and a total climb of 90 feet on $[0, 1]$. This gives $r'(t) =< -120\pi \sin 6\pi t, 120\pi \cos 6\pi t, 90 >$. The force due to gravity is a constant 160+25=185 pounds straight down, which is $< 0, 0, -185 >$. The force the man exerts is $F =< 0, 0, 185 >$.
    The work is now $\int_0^1 < 0, 0, 185 > \cdot < 120\pi \cos 6\pi t, 120\pi \sin 6\pi t, 90 > dt = \int_0^1 185(90)dt = 16650$ foot-pounds.
    Notice that the x- and $y-$components of the force did not appear in the integral. This is because gravity only acts vertically, so whatever work the man does against gravity is in a vertical direction, too. Therefore, we could have just said, "Oh! He's raising 185 pounds 90 feet, which is $185(90) = 16650$ foot-pounds of work!"