Solutions to Homework Assignment 31

MATH 249-01 and -02

Section 16.5, Page 1096

1, 3, 5, 7, 9, 10, 11, 12, 13, 15, 17

1. (a) $\nabla \times F = < -x^2, 3xy, -xz >$.
   (b) $\nabla \cdot F = yz + 0 + 0 = yz$.
3. (a) $\nabla \times F = < x - y, -y, 1 >$.
   (b) $\nabla \cdot F = 0 + z - \frac{1}{2\sqrt{z}} = z - \frac{1}{2\sqrt{z}}$. Note that the answer in the back of the book is wrong.
5. (a) $\nabla \times F = < 0, 0, 0 >$.
   (b) $\nabla \cdot F = e^x \sin y - e^x \sin y + 1 = 1$.
7. (a) $\nabla \times F = < 1/y, -1/x, 1/x >$.
   (b) $\nabla \cdot F = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

For 9, 10, and 11, I will assume F has the form $F = < P, Q, 0 >$.

9. Notice that F does not change as we move left to right, so $P = 0$ and $F = < 0, Q, 0 >$. However, F gets shorter and shorter as y increases, so $Q_y < 0$. Thus div $F = Q_y < 0$. For the curl, we need only consider the z-component, which is $Q_x - P_y$. (This is because F is independent of z.) Since F and Q are independent of x, $Q_x = 0$, so curl $F = 0$.
10. Here, $P_x > 0$ and $Q_y > 0$ since the vectors get longer the father we get from the origin. Thus div $F = P_x + Q_y > 0$. Notice that as we move to the right, the y-components of the vectors do not change. This means that $Q_x = 0$. Also, as we move up, the x-components do not change. Therefore, $Q_x - P_y = 0$, so curl $F = 0$.
11. As we move from left to right, the x-components of the vectors remain constant, so $P_x = 0$. As we move up, the x-components increase, so $P_y > 0$. The y-components are all zero, so $Q = 0$. Now div $F = 0$ and curl $F = < 0, 0, Q_x - P_y > = < 0, 0, -P_y >$. Since $P_y > 0$, this points down.
12. (a) This is meaningless; the curl only applies to vector fields, not scalar functions.
    (b) This is a vector field; its components are the first partial derivatives of f.
    (c) This is a scalar field.
    (d) This is a vector field.
    (e) This is meaningless; the gradient only applies to scalar functions.
    (f) This is a vector field.
    (g) This is a scalar field.
    (h) This is meaningless; div only applies to vector fields.
    (i) This is a vector field.
    (j) This is meaningless; div only applies to vector fields and div F is a scalar field.
    (k) This is meaningless; div F is a scalar field, so we can't cross it with anything.
    (l) This is a scalar field.
13. curl $F = < x - x, -(y - y), z - z > = < 0, 0, 0 >$. Therefore, the vector field is conservative. If $\nabla f = F$, then $f_x = yz$, so $f = xyz + g(y, z)$. Also, $f_y = xz$ implies $f = xyz + h(x, z)$ and $f_z = xy$ implies $f = xyz + k(x, y)$. We may take $f(x, z, y) = xyz$.
15. curl $F = < 2y - 2y, -(0 - 0), 2x - 2x > = < 0, 0, 0 >$. Therefore, F is conservative. Now $f_x = 2xy$, so $f = x^2y + g(y, z)$. With $f_y = x^2 + 2yz$, we get $f = x^2y + y^2z + h(x, z)$, and with $f_z = y^2$, we get $f = y^2z + k(x, y)$. Comparing these, we see that $f(x, y, z) = x^2y + y^2z$ will work.
17. curl $F = < 0 - 0, -(0 - 0), -e^{-x} - e^{-x} > = < 0, 0, -2e^{-x} >$, so F is not conservative.