

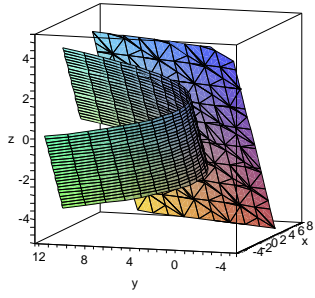
## Solutions to Homework Assignment 32

MATH 249

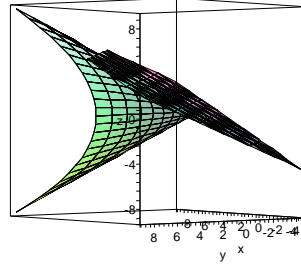
Section 16.6 Stewart 6e, Page 1078

3-6, 11-16, 21, 23, 33, 34, 35, 47

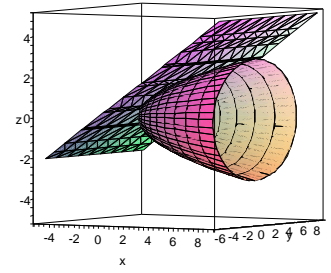
3. This surface is the graph of a plane through  $(0, 3, 1)$  that contains the vectors  $\langle 1, 0, 4 \rangle$  and  $\langle 1, -1, 5 \rangle$ . (Notice that the components are all linear in  $u$  and  $v$ .)
4. Notice that  $(x/2)^2 + (y/3)^2 = 1$ , so this graph is an elliptical cylinder.  $z$  can take on any value (and  $x$  does not appear in the cylinder equation), so the cylinder has the  $z$ -axis as its axis.
6. This surface satisfies  $x^2 + z^2 = y$ , so the graph is a circular paraboloid.
13. This isn't one of our standard quadratic surfaces. Notice, though, that as the value of  $v$  increases, the height increases and the angle of the component in the  $xy$ -plane increases. This is the spiral ramp (helicoid) in IV.
14. This should have cylindrical symmetry, so it is not II, III, or V. If we consider the grid curve  $v = 0$ , we get  $z = \sin(x)$ , so a side view should look sinusoidal. This matches with I.
15. This one has cylindrical symmetry centered on the  $x$ -axis. Only II has that property.
16. If we hold  $v$  constant, we get  $(x, y)$  following an outward spiral – the  $\cos 4\pi u$  and  $\sin 4\pi u$  move the point around the  $x$ -axis, and the factor of  $(1 - u)(3 + \cos v)$  that both  $x$  and  $y$  have increases the radius. Also,  $z$  will increase as  $u$  increases.  
When  $u$  is held constant, we have something of the form  $x = \alpha_1 + \beta_1 \cos v$ ,  $y = \alpha_2 + \beta_2 \cos v$ , and  $z = \alpha_3 + \beta_3 \sin v$ . These give ellipses in the  $xz$ - and  $yz$ -planes and, in fact, all the way around the  $z$ -axis. This matches V.
17. This is an interesting one! Note that  $x^{3/2} + y^{3/2} + z^{3/2} = 1$ . Thus, this should be symmetric in  $x$ ,  $y$ , and  $z$ . Only III matches that.
18. Consider the grid curve  $v = 0$ . We get  $x = 1 - |u|$ ,  $y = 0$ ,  $z = u$ . The cross section is  $x = 1 - |z|$ , which matches VI.
21. To the right of the  $xz$ -plane,  $y > 0$ , so we may solve:  $y = \sqrt{1 + z^2 - x^2}$ . Now take  $x$  and  $z$  as parameters.
23. The cone has spherical equation  $\phi = \pi/4$ . We have seen that  $x = 2 \sin \phi \cos \theta$ ,  $y = 2 \sin \phi \sin \theta$ ,  $z = 2 \cos \phi$  will give a parametrization of sphere of radius 2 centered at the origin; we just need to take  $\phi \in [0, \pi/4]$  to restrict this to lie above the cone.
33.  $r = \langle u + v, 3u^2, u - v \rangle$ . To get  $\langle x, y, z \rangle = \langle 2, 3, 0 \rangle$ , we need  $u = v = 1$ .  $r_u = \langle 1, 6u, 1 \rangle$ ,  $r_v = \langle 1, 0, -1 \rangle$ , and at our point we have  $r_u(1, 1) = \langle 1, 6, 1 \rangle$ ,  $\langle 1, 0, -1 \rangle$ , and  $r_u \times r_v = \langle -6, 2, -6 \rangle$ . I will use  $\langle -3, 1, -3 \rangle$ . The plane therefore has equation  $-3(x-2) + (y-3) - 3z = 0$ , or  $3x - y + 3z = 3$ .
34. (For a little extra practice)  $r = \langle u^2, v^2, uv \rangle$ , so  $r_u = \langle 2u, 0, v \rangle$  and  $r_v = \langle 0, 2v, u \rangle$ . Therefore  $r_u \times r_v = \langle -2v^2, -2u^2, 4uv \rangle$ . At  $(u, v) = (1, 1)$ , this is  $\langle -2, -2, 4 \rangle$ , and  $r(1, 1) = \langle 1, 1, 1 \rangle$ . Our tangent plane has equation  $-2(x-1) - 2(y-1) + 4(z-1) = 0$ , or  $2z = x + y$ . The graphs are below.
35.  $r(1, 0) = \langle 1, 0, 1 \rangle$ .  $r_u = \langle 2u, 2 \sin v, \cos v \rangle$ ,  $r_v = \langle 0, 2u \cos v, -u \sin v \rangle$ , and  $r_u \times r_v = \langle -2u^2, 2u^2 \sin v, 4u^2 \cos v \rangle$ . At  $(u, v) = (1, 0)$ , the normal is thus  $\langle -2, 0, 4 \rangle$ . I will use  $\langle -1, 0, 2 \rangle$ . We get the equation  $-(x-1) + 2(z-1) = 0$ , or  $2z - x = 1$ .



Number 31



Number 32



Number 33