Solutions to Homework Assignment 34

MATH 249

Section 16.9 Stewart 6e, Page 1103

1, 5, 6, 7, 9, 11, 12, 14, 19

1. To compute the surface integral directly, we need to compute 6 different surface integrals. Fortunately, there are a lot of zeros involved! Other good news: since the surface is made up of faces parallel to the planes, we will have $dS = dA$ in each case.
   In the plane $x = 0$, we will just get zero since $F =< 0, 0, 0 >$ in this plane. In the plane $x = 1$, the outward unit normal is $< 1, 0, 0 >$ and $F =< 3, y, 2z >$, so we have $\iint_S F \cdot d\vec{S} = \iint_S F \cdot \hat{n} dS = \int_0^1 \int_0^1 3dydz = 3$.
   In the plane $y = 0$, we have $F =< 3x, 0, 2xz >$ and $\hat{n} =< 0, -1, 0 >$, so $F \cdot \hat{n} 0$ and the corresponding integral is 0. In the plane $y = 1$, we have $F =< 3x, x, 2xz >$ and $\hat{n} =< 0, 1, 0 >$, so $\iint_S F \cdot \hat{n} dS = \int_0^1 \int_0^1 xdzdx = \frac{1}{2}$.
   In the plane $z = 0$, we have $F =< 3x, xy, 0 >$ and $\hat{n} =< 0, 0, -1 >$, so $F \cdot \hat{n} = 0$ and the corresponding integral is again 0. In the plane $z = 1$, we get $F =< 3x, xy, 2x >$ and $\hat{n} =< 0, 0, 1 >$, so $\iint_S F \cdot \hat{n} dS = \int_0^1 \int_0^1 2xdydx = 1$.
   All together, we have $3 + \frac{1}{2} + 1 = \frac{9}{2}$. Whew!
   Now let's apply the divergence theorem. $\text{div}F = 3 + x + 2x = 3 + 3x$, so $\iint_S F \cdot d\vec{S} = \int_0^1 \int_0^1 \int_0^1 3 + 3xdzdydx = \frac{9}{2}$. Yeah, baby!
5. We get $\int_0^1 \int_0^1 \int_0^2 (e^x \sin y - e^x \sin y + 2yz) dzdydx = \int_0^1 \int_0^1 4ydydx = 2$.
6. $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 (2xz^3 + 2xz^3 + 4xz^3) dzdydx = 4 \int_{-1}^1 \int_{-3}^3 8xz^3 dzdx = 4 \int_{-1}^1 0dx = 0$.
7. $\text{div}F = 3y^2 + 0 + 3z^2 = 3y^2 + 3z^2$. If we let $y = r\cos\theta, z = r\sin\theta, x = x$ (modified cylindrical coordinates), we get $\int_{-1}^2 \int_0^{2\pi} \int_0^1 3r^2(r) drd\theta dx = 18\pi \int_0^1 r^3 = \frac{9}{2}$.
9. $\text{div}F = y\sin z + 0 - y\sin z = 0$, so the integral is just 0. Whew! We would have had a tough time with the ellipsoid!
11. $\text{div}F = y^2 + 0 + x^2 = x^2 + y^2$. I will convert to cylindrical coordinates for the integral, getting $\int_0^{2\pi} \int_0^2 \int_{r^2}^4 r^2(r) dzdrd\theta = 2\pi \int_0^2 \int_{r^2}^4 r^3 dzdr = 2\pi \int_0^2 4r^3 - r^5 dr = \frac{32\pi}{3}$.
12. $\text{div}F = 4x^3 + 0 + 4xy^2 = 4x(x^2 + y^2)$. In cylindrical coordinates, we get $\int_0^{2\pi} \int_0^1 \int_0^{r\cos\theta + 2} 4r\cos\theta(r^2) rdzdrd\theta = \int_0^{2\pi} \int_0^1 4r^4 \cos\theta(r\cos\theta) drd\theta = \int_0^{2\pi} \cos^2\theta d\theta \int_0^1 4r^5 dr = \int_0^{2\pi} \frac{1}{2}(1 + \cos 2\theta) d\theta \frac{2}{3} = \frac{2\pi}{3}$.
19. More "stuff" seems to be entering P_1 than leaving it, so the divergence of F at P_1 is negative. At P_2, the situation is opposite, so the divergence at P_2 is positive.