MATH 249

Today

1. Questions from last time
2. 12.4: The cross product. (Understand calculation, interpretation, and applications)
3. WeBWorK
4. Homefun 2

12.4 Cross Product

Used for:

- Physics (Torque, electricity and magnetism, curl, angular momentum...)

Let $\vec{u} =< u_1, u_2, u_3 >, \vec{v} =< v_1, v_2, v_3 >$, and $\vec{w} =< w_1, w_2, w_3 >$.

1. $\vec{u} \times \vec{v} =< u_2v_3 - u_3v_2, -(u_1v_3 - u_3v_1), u_1v_2 - u_2v_1 >$ (12-04a: 0:00. For derivation: 12-04b: 0:00)
2. $\vec{u}$ and $\vec{v}$ are orthogonal to $\vec{u} \times \vec{v}$. (12-04a: 2:33))
3. The cross product is **only** defined for 3-dimensional vectors. We can fake it for 2D vectors, though. (12-04a: 2:51)
4. Example (12-04a: 3:23)
5. The triple $\vec{u}, \vec{v}, \vec{u} \times \vec{v}$ obeys the Right-Hand Rule. (12-04a: . For proof: 12-04b: 6:32.)
6. $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta$, where θ is the angle between $\vec{u}$ and $\vec{v}$. (12-04a" 9:35. For proof: 12-04b: 12:32)
7. $\vec{u}$ and $\vec{v}$ are parallel if and only if $\vec{u} \times \vec{v} = \vec{0}$. (12-04a: 11:00)
8. The area of the parallelogram determined by $\vec{u}$ and $\vec{v}$ is $|\vec{u} \times \vec{v}|$. (12-04a: 12:12)
9. Example (12-04a: 13:27)
10. Determinant method (Optional. Good for physics students. Can skip to 20:07.) (12-04a: 17:30)

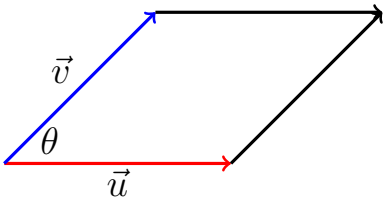
11. Algebraic properties (12-04a: 20:07)

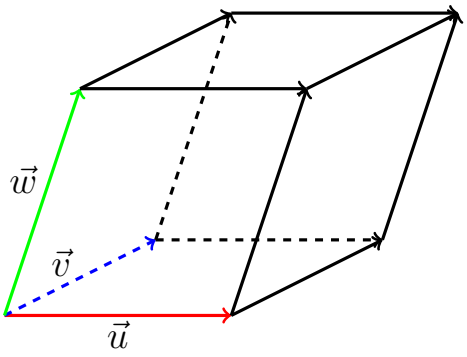
    (a) $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$.

    (b) The cross product is neither associative nor commutative.

    (c) $(c\vec{u}) \times \vec{v} = c(\vec{u} \times \vec{v}) = \vec{u} \times (c\vec{v})$ for any scalar c.

    (d) $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$ and $(\vec{u} + \vec{v}) \times \vec{w} = \vec{u} \times \vec{w} + \vec{v} \times \vec{w}$.





12. The volume of the parallelepiped determined by $\vec{u}, \vec{v}$, and $\vec{w}$ is $V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$. (12-04a: 23:00. For proof: 12-04b: 15:30)

13. Physics examples. (12-04a: 23:49)

14. Examples: p. 792: 1-7, 9, 13, 16, 28, 35

15. WeBWorK: 1, 5, 8, 9, 10, 11

Next Time

1. Watch 12-05-10-01 and 12-05-lines-planes-b [$\sim$ 25 and 27 minutes, respectively]