MATH 249

Today

1. 14.4 Tangent planes and linearization
   - Understand tangent planes as a generalization of tangent lines.
   - Know how to find equations of tangent planes.
   - Understand that tangent planes can be used to approximate function values near the point of tangency
   - Understand differentials as an approximation of the change in a function
2. WeBWorK

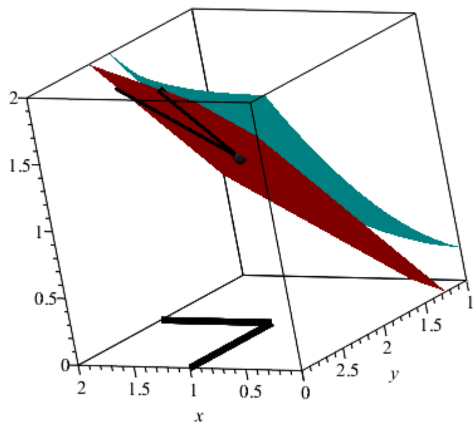
14.4 Tangent Planes

1. Tangents in the x and y directions to a surface at a point $P(x_0, y_0, z_0)$ define a plane.
2. The normal to this plane is given by

   $$< 1, 0, f_x(x_0, y_0) > \times < 0, 1, f_y(x_0, y_0) > = < -f_x(x_0, y_0), -f_y(x_0, y_0), 1 > .$$

3. Thus, the tangent plane has equation

   $$z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$



[See also Maple file 14-04 for other views.]

4. The **Linearization** of f at (x_0, y_0) is

$$L(x, y) = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

5. The graph of the linearization of f is the tangent plane to f at (x_0, y_0, z_0). NOTE: the only places x and y appear are in $x - x_0$ and $y - y_0$. The rest are constants x_0 and y_0!

6. Let $z = f(x, y)$. Then f is **differentiable** at (x_0, y_0) if

$$\Delta z = f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

for some $\epsilon_1, \epsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0, 0)$.

7. **Differentials**: $dz = f_x(x, y)dx + f_y(x, y)dy$, where $dx = \Delta x = x - x_0$ and $dy = \Delta y = y - y_0$.

8. Thus, for a differentiable function, we have $\Delta z = dz(x_0, y_0) + \epsilon_1 dx + \epsilon_2 dy$, so $\Delta z \approx dz$.

9. Put another way: if f is a differentiable function, then the tangent plane is a good approximation of the graph of f – and the linearization is a good approximation of the function – near (x_0, y_0).

10. **Theorem**: If f_x and f_y exist near (x_0, y_0) and are continuous at (x_0, y_0), then f is differentiable at (x_0, y_0).

11. Comparisons:

    (a) $L(x, y) = z_0 + dz(x_0, y_0)$.

   (b) The tangent plane is given by $z = L(x, y)$.
   (c) $\Delta z = dz +$ a small error term (if f is differentiable).
12. Examples p. 899: #3, 14, 29, 37, 34
13. WeBWorK: 2, 4

Next Time

1. Watch 14.5 [$\sim$ 35 minutes]