MATH 249

Today

1. 14.5 The Multivariable Chain Rule (Understand how to compute partial derivatives of compositions.)

2. WeBWorK

14.5 Multivariable Chain Rule

1. The Chain Rule: If $z = f(x, y)$ is a differentiable function of x and y and $x = g(t), y = h(t)$ are differentiable functions of t, then z is a differentiable function of t, and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}.$$

   If, instead, $x = g(s, t), y = h(s, t)$ are differentiable functions of s and t, then z is a differentiable function of s and t, and

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \text{and } \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}.$$

2. This also generalizes to more variables – see page 904.

3. Implicit Differentiation: Assume that all partial derivatives involved are continuous near the point of interest. Then:

   (a) If $F(x, y) = C$, then $\frac{dy}{dx} = -\frac{F_x}{F_y}$.

   (b) If $F(x, y, z) = C$, then $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$ and $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$.

4. Examples p. 907: #1, 9, 13, 15, (23), 34, 37, (33)

5. WeBWorK: 2, 4

Next Time

1. Watch 14.6 [$\sim$ 49 minutes] – Very important section!