# MATH 249

## Today

1. Optimization Summary

## Optimization Summary

1. Multivariable optimization has strong similarities to single-variable optimization.
   (a) Local extrema can only occur at critical points.
   (b) Global extrema are guaranteed for continuous functions on regions that are closed and bounded. [Extreme Value Theorem]
   (c) In such cases, the global extrema must occur either at a critical point or a point on the boundary.

2. Local optimization:
   (a) Critical points occur where $f_x$ and $f_y$ are both 0 or undefined: $(a, b)$ is a critical point if $f_x(a, b) = 0 = f_y(a, b)$ or at least one is undefined.
   (b) Second Derivatives Test: Let $D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2$.
      i. If $D(a, b) > 0$, then $f$ has a local extremum at $(a, b)$. Max if $f_{xx}(a, b) < 0$, and min if $f_{xx}(a, b) > 0$.
      ii. If $D(a, b) < 0$, then $f$ has a saddle point at $(a, b)$.
      iii. If $D(a, b) = 0$, then the Second Derivatives Test gives no information.

3. Global optimization:
   (a) We still need critical points.
   (b) We also need to consider the behavior of $f$ on the boundary, where it becomes a function of one variable and we can optimize as in Calc I.
   (c) Out of all of the candidates found in (a) and (b), the largest is the global max and the smallest is the global min.

4. Lagrange multipliers for constrained optimization:
   (a) This is another technique for global optimization, but on a curve rather than a region. Thus, it can be applied to the boundary of a global optimization problem.
   (b) The maximum and minimum values of $f(x, y)$ when $(x, y)$ is constrained to lie on a curve $g(x, y) = C$ can only occur where $\nabla f = \lambda \nabla g$ for some scalar $\lambda$.
   (c) To find such candidates, solve the system

$$\begin{aligned} f_x &= \lambda g_x \\ f_y &= \lambda g_y \\ g(x, y) &= C \end{aligned}$$

   for $x, y$, and $\lambda$. More equations required for more variables.
   (d) Evaluate $f$ at each candidate. The largest output is the max, and the smallest is the min.
   (e) Note that this technique does not require critical points (although you may also need them if the problem isn't confined to a curve).

5. Word problems
   (a) The first step is to create a model for the problem. Take advantage of any symmetry, physical constraints, etc.
   (b) Once you have a model, identify appropriate techniques: is it a local optimization problem? Global? Constrained?
   (c) Solve the mathematical problem you have translated to.
   (d) Translate the results back to interpret them in context.

## Next Time

1. Review for Exam

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