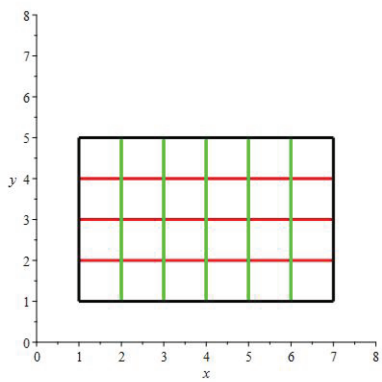
MATH 249

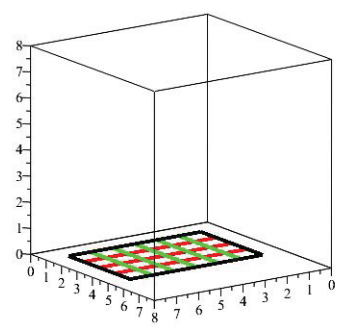
Today

1. Go over exam
2. 15.1: Double Integrals (Understand the definition and interpretation of definite integrals.)

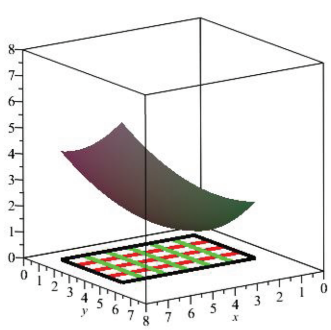
15.1 Double Integrals

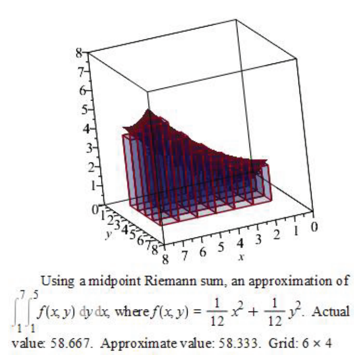
1. Let S be a surface given by $z = f(x, y)$ over a rectangle $R = [a, b] \times [c, d]$.
2. Subdivide $[a, b]$ into m subintervals $[x_{i-1}, x_i]$ of equal width Δx and $[c, d]$ into n subintervals $[y_{j-1}, y_j]$ of width Δy.
3. This subdivides R into mn subrectangles $R_{ij} = [x_{i-1}, x_i] \times [y_{j-1}, y_j]$ of equal area $\Delta A = \Delta x \Delta y$.





4. Choose a sample point (x_{ij}^*, y_{ij}^*) from each R_{ij} and construct a box with base R_{ij} and height $f(x_{ij}^*, y_{ij}^*)$.





5. For a positive function f on R, the volume of such a box is $f(x_{ij}^*, y_{ij}^*)\Delta A$.

6. Thus, an approximation for the volume is

$$\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A.$$

7. But we don't want an approximation! The volume is

$$V = \lim_{m,n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A = \iint\limits_R f(x,y)dA,$$

   provided this limit exists.

8. **Theorem:** If f is continuous on R, then $\iint\limits_R f(x,y)dA$ exists.

9. Notes:

   (a) We do not need f positive for this definition to make sense.

   (b) We will repeat this subdivide-and-conquer approach many times this semester.

   (c) Right now, we have **no** techniques for evaluating these except for geometry. We'll get some, though.

10. Examples p. 940: #3, 5, 8, 10, 12

11. If time: try the **iterated integral** $\int_1^3 \int_0^1 (-x-4y)dxdy$ (three ways).

12. **Theorem (Fubini's Theorem)**: If f is continuous on $R = [a,b] \times [c,d]$, then

$$\iint\limits_R f(x,y)dA = \int_a^b \int_c^d f(x,y)dydx = \int_c^d \int_a^b f(x,y)dxdy.$$

Next Time

1. Watch 15.2 [$\sim$ 27 minutes] and start 15.3 [first $\sim$ 35 minutes]