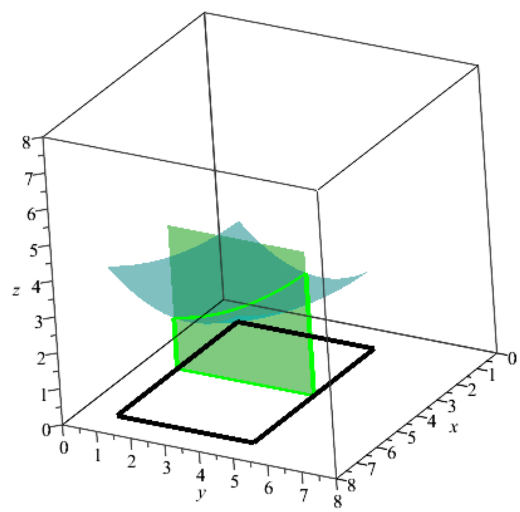
MATH 249

Today

1. 15.2: Iterated Integrals (Understand the definition of an iterated integral and how they differ from double integrals. Understand Fubini's Theorem.)

15.2 Iterated Integrals

1. Let $A(x) = \int_c^d f(x,y)dy$. Then $A(x)$ is a **partial integral** of f with respect to y.



   $A(x)$ represents the cross-sectional area under the graph of f for a fixed value of x.

2. $\int_a^b A(x)dx = \int_a^b \left(\int_c^d f(x,y)dy \right) dx = \int_a^b \int_c^d f(x,y)dydx.$

3. Similarly, $\int_c^d \int_a^b f(x,y)dxdy = \int_c^d \left(\int_a^b f(x,y)dx \right) dy.$

4. Notes:

   (a) We need the dx and dy so we know the variable of integration. Keep track of their order!

   (b) Integration proceeds from the "inside" out.

(c) These integrals are called **iterated integrals**.

5. **Fubini's Theorem**: If f is continuous on $R = [a, b] \times [c, d]$, then

$$\iint_R f(x,y)dA = \int_a^b \int_c^d f(x,y)dydx = \int_c^d \int_a^b f(x,y)dxdy.$$

6. Notice that even though double integrals and iterated integrals are defined in completely different ways, Fubini's Theorem says they come out the same as long as f is continuous.

7. Iterated integrals can be thought of as finding the volume be adding up volumes of infinitesimally thick slabs, so it makes sense that the total volume would be the same as it was for double integrals.

8. Actually, f doesn't have to be continuous – just "close enough" to continuous.

9. **Special Case**: If $f(x, y) = g(x)h(y)$ (f can be factored into a function of x alone times a function of y alone), then

$$\begin{aligned}\iint_R f(x,y)dA &= \int_a^b \int_c^d g(x)h(y)dydx \\ &= \int_a^b g(x)\left(\int_c^d h(y)dy\right)dx \\ &= \left(\int_a^b g(x)dx\right)\left(\int_c^d h(y)dy\right).\end{aligned}$$

10. Examples p. 964: #7, 5, 13

Next Time

1. Watch 15.3 [the rest]