MATH 249

Today

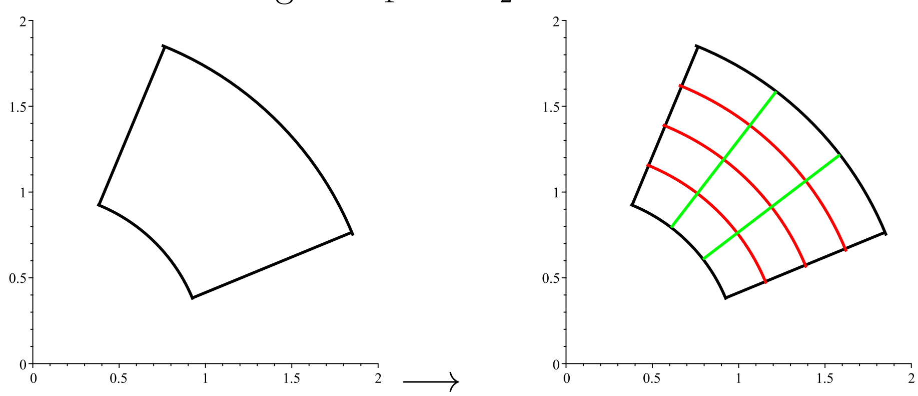
1. 15.4: Double Integrals in Polar Coordinates (Understand how to convert to and integrate in polar coordinates.)
2. WeBWorK
3. Homefun/Python

15.4: Double Integrals in Polar Coordinates

1. Recall: in polar coordinates, $x = r\cos(\theta), y = r\sin(\theta), x^2 + y^2 = r^2$, and $\dfrac{y}{x} = \tan\theta$.
2. The area of a polar rectangle $[r_1, r_2] \times [\theta_1, \theta_2]$ is

$$\pi(r_2^2 - r_1^2)\frac{\theta_2 - \theta_1}{2\pi} = \frac{1}{2}(r_2 + r_1)\Delta r\Delta\theta = r^*\Delta r\Delta\theta = \Delta A,$$

where r^* is the average of r_1 and r_2.

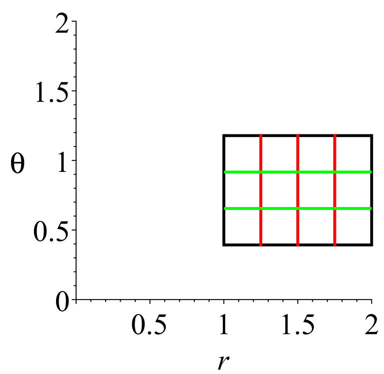


If we choose that average as the sample point from each subrectangle, our volume calculation (from the definition of the double integral) over the rectangle $[a, b] \times [\alpha, \beta]$ becomes

$$\begin{aligned} V &= \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A \\ &= \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(f(r_{ij}^* \cos(\theta_{ij}^*), r_{ij}^* \sin(\theta_{ij}^*)) r_{ij}^*\right) \Delta r \Delta\theta \\ &= \int_{\alpha}^{\beta} \int_{a}^{b} f(r\cos(\theta), r\sin(\theta)) r\,dr\,d\theta, \end{aligned}$$

where the iterated integral is over a rectangle in the $r\theta$-plane. Compare to the definition of the definite integral:

$$\iint\limits_R f(x,y)dA = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta x \Delta y$$



3. Note that $A(R) = \iint\limits_R 1dA$.
4. Examples p. 978: #1-4, 5, 6, 10, 11, 15, 18, 19, 22, 24, 29, 32
5. WeBWorK: 4, 5, 6

Next Time

1. Watch 15.6 [$\sim$ 50 minutes]
2. Homefun 10