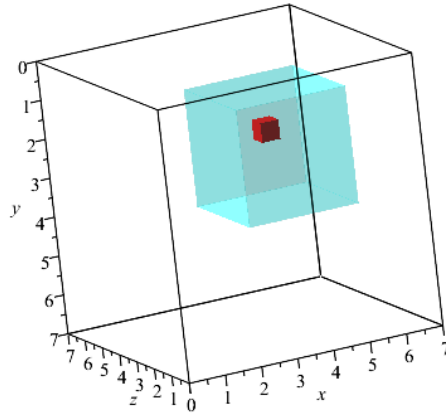


Today

1. 15.6: Triple Integrals (Understand the definition and computation of triple integrals (Fubini's Theorem). Compare to double integrals.
2. WeBWorK
3. Homefun

15.6: Triple Integrals

1. Think of $f(x, y, z)$ as representing density. We want the mass of a solid box $E = [a, b] \times [c, d] \times [r, s]$.
2. Subdivide E into subboxes E_{ijk} by subdividing $[a, b]$ into ℓ subintervals of width Δx , $[c, d]$ into m intervals of width Δy , and $[r, s]$ into n subintervals of width Δz . The i , j , and k refer to the subintervals associated with the box: the i th subinterval in the x direction, the j th in the y , and the k th in the z .



3. There are then lmn subboxes of volume $\Delta V = \Delta x \Delta y \Delta z$.
4. From each subbox, choose a sample point $P_{ijk}^* = (x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)$.
5. If the subboxes are small enough, the $f(P_{ijk}^*)$ is a pretty good approximation for the density of the whole subbox (assuming f is continuous). Thus, the mass of a subbox is approximately $f(P_{ijk}^*)\Delta V$.
6. Now the mass of E is approximately the sum of the masses of the subboxes:

$$\sum_{i=1}^{\ell} \sum_{j=1}^m \sum_{k=1}^n f(P_{ijk}^*)\Delta V.$$

7. As usual, we don't want an approximation; we want an exact value! So we take a limit and define the **triple integral of f over E** :

$$\iiint_E f(x, y, z) dV = \lim_{\ell, m, n \rightarrow \infty} \sum_{i=1}^{\ell} \sum_{j=1}^m \sum_{k=1}^n f(P_{ijk}^*)\Delta V,$$

provided the limit exists.

8. Note that the definition of the triple integral doesn't depend on f being positive (i.e., representing density).
9. **Fubini's Theorem:** If f is continuous on $B = [a, b] \times [c, d] \times [r, s]$, then

$$\iiint_B f(x, y, z) dV = \int_a^b \int_c^d \int_r^s f(x, y, z) dz dy dx$$

in any order.

10. We can also integrate over non-box regions as we did over non-rectangular regions in the plane for double integrals.
11. The volume of a region E in \mathbb{R}^3 is $\iiint_E 1dV$.
12. Examples p. 998: #6, 11, 14, 19, 22, 27-28, 53
13. WeBWorK: 4, 5, 6

Next Time

1. Watch 15.7-8 [\sim 45 minutes]