MATH 249

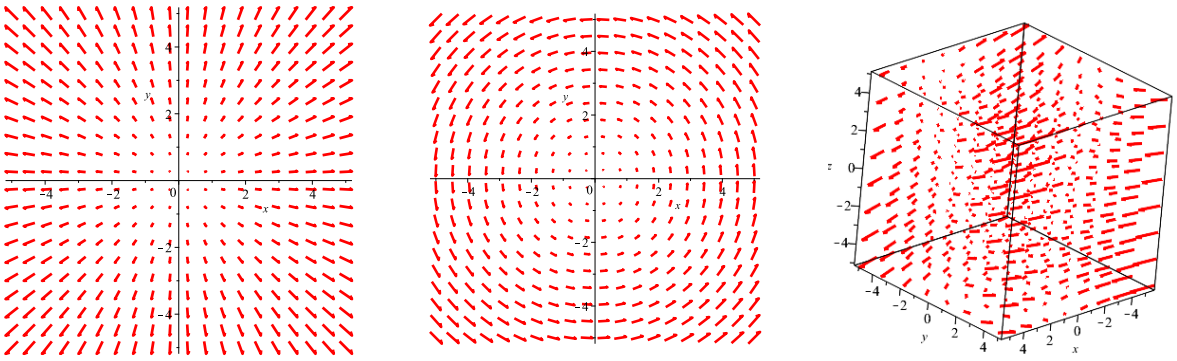
Today

1. 16.1: Vector Fields (Understand the definition and graph of a vector field.)
2. 16.2: Line integrals (Understand the definition and computation of a scalar path integral.)
3. WeBWorK
4. Homefun/Python

16.1: Vector Fields

1. A **vector field** is a function $\vec{F} : \mathbb{R}^n \to V_n$.
2. The input to a vector field is a point in n-dimensional space, and the output is an n-dimensional vector: a vector at each point in space.

3.



4. A vector field $\vec{F}$ is called **conservative** if $\vec{F} = \nabla f$ for some scalar function f. In this case, f is called a **potential function** of $\vec{F}$.

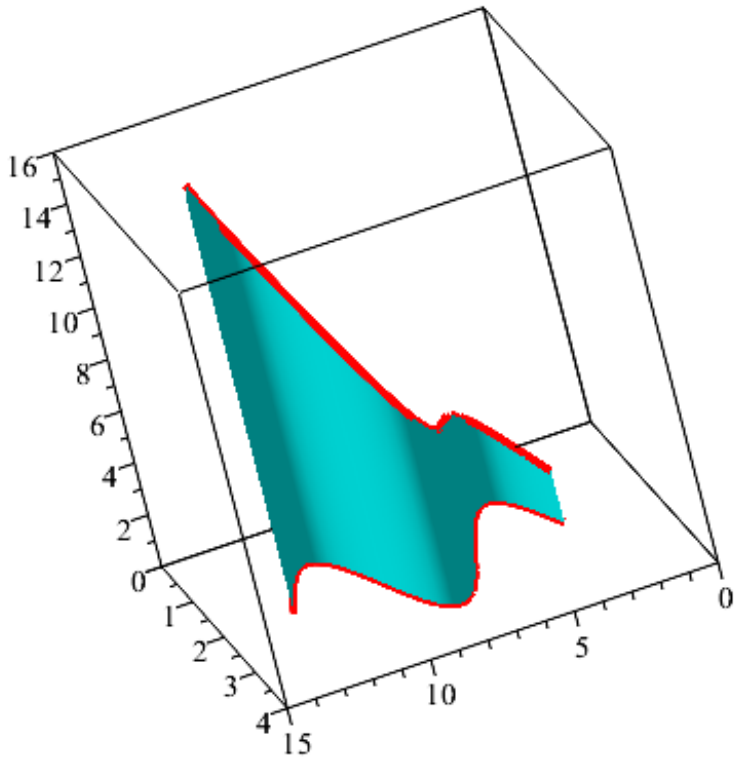
16.2: Line Integrals (aka Path Integrals)

5. Let C be a curve parametrized by $\vec{r}(t)$ for $t \in [a, b]$, and let f be a function whose domain includes C. The **path integral** of f over C is

$$\int_C f(x,y)ds = \lim_{n\to\infty} \sum_{i=1}^{n} f(x,y)\Delta S_i = \int_a^b f(r(\vec{t}))|\vec{r}\,'(t)|dt.$$

6. Note that $ds = |\vec{r}\,'(t)|dt$.

7. We can think of this path integral as the area of a "shower curtain" lying above C and below the graph of f.



8. We also define $\int_C f(x,y)dx = \int_a^b f(\vec{r}(t))x'(t)dt$ and $\int_C f(x,y)dy = \int_a^b f(\vec{r}(t))y'(t)dt$
9. Examples p. 1032: #1-10, 11-14, 29-32, 35a
10. Examples p. 1043: #2, ...
11. Python

Next Time

1. Watch the rest of 16.2 [$\sim$43 minutes]
2. Homefun