MATH 249

Today

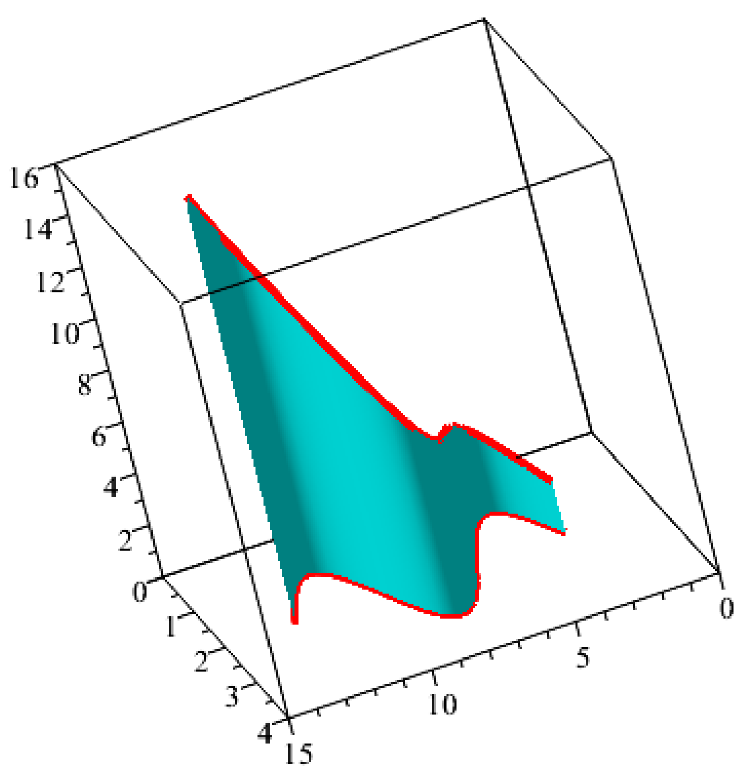
1. 16.2: Line integrals (Understand and be able to compute path integrals of vector fields.)
2. WeBWorK
3. Homefun 12

16.2: Line Integrals (aka Path Integrals)

1. Let C be a curve parametrized by $\vec{r}(t)$ for $t \in [a, b]$, and let f be a function whose domain includes C. The **path integral** of f over C is

$$\int_C f(x,y)ds = \lim_{n\to\infty}\sum_{i=1}^{n} f(x,y)\Delta S_i = \int_a^b f(r(\vec{t}))|\vec{r}\,'(t)|dt.$$

2. Note that $ds = |\vec{r}\,'(t)|dt$.
3. We can think of this path integral as the area of a "shower curtain" lying above C and below the graph of f.



4. $\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}dt$

5. $\int_C f(x,y)dx = \int_a^b f(x(t),y(t))x'(t)dt$

6. $\int_C f(x,y)dy = \int_a^b f(x(t),y(t))y'(t)dt$

7. Recall that the **work** done by a force $\vec{F}$ along a displacement D is given by $W = \vec{F}\cdot\vec{D}$.

8. Suppose $C : \vec{r}(t)$ is smooth on $[a,b]$. Subdivide $[a,b]$ into n subintervals of width Δt. This also divides C into n subarcs, where the ith subarc has length Δs_i and moves in the direction of $\vec{r}\,'(t_i)$. Thus, the work done by $\vec{F}$ in moving along this little bit of the path is approximately $\vec{F}(\vec{r}(t_i))\cdot(\vec{r}\,'(t_i))\Delta s_i$.

9. As before, $\Delta s_i = |\vec{r}\,'(t_i)|\Delta t$.

10. Let C be a smooth curve parametrized by $\vec{r}$ on $[a,b]$, and let $\vec{T}$ be the unit tangent vector $\dfrac{\vec{r}\,'}{|\vec{r}\,'|}$. If $\vec{F} =< P,Q >$ is a continuous vector field on C, we define the **line** (or **path**) **integral of** $\vec{F}$ **along** C by

$$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)dt = \int_C \vec{F}\cdot\vec{T}ds = \int_C Pdx + Qdy.$$

11. Recall that the scalar projection of $\vec{F}$ onto $\vec{T}$ is given by $\text{comp}_{\vec{T}}\vec{F} = \dfrac{\vec{F}\cdot\vec{T}}{|\vec{T}|} = \vec{F}\cdot\vec{T}$, the integrand. That is, we are adding up the contribution of $\vec{F}$ to traveling along the path.

12. Examples p. 1043: #2, 3, 5, 6, 8, 17, 22, 42, 43

13. Python

Next Time

1. Watch 16.3 [∼51 minutes]