MATH 249

Today

1. 16.5: Curl and Divergence (Understand the computation and interpretation of the curl and divergence of a vector field and the del operator.)
2. WeBWorK
3. Homefun/Python

16.5: Curl and Divergence

1. We define the **operator** "del" by $\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$.
2. ∇ is not a "vector" in our usual sense since its components are not numbers, but rather instructions to differentiate.
3. Notice that the gradient of f is $\nabla f = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle =< f_x, f_y, f_z >$, as though it were scalar multiplication.
4. The **curl** of the vector field $\vec{F} =< P, Q, R >$ is
$$\text{curl}\vec{F} =< R_y - Q_z, P_z - R_x, Q_x - P_y >= \nabla \times \vec{F}.$$
5. Properties:
   (a) $\text{curl}(\nabla f) = \vec{0}$ if f has continuous second-order partials.
   (b) If $\vec{F}$ is conservative, then $\text{curl}\vec{F} = \vec{0}$. (From (a).)
   (c) Test for Conservatism: If $\vec{F}$ is a vector field on a simply connected region D, $\vec{F}$ has continuous partials on D, and $\text{curl}\vec{F} = \vec{0}$ on D, then $\vec{F}$ is conservative.
   (d) If $\text{curl}\vec{F} = \vec{0}$, we call $\vec{F}$ **irrotational**.
   (e) The curl is a measure of how much the vector field wants to rotate about a point. Its magnitude is the velocity of that rotation, and its direction is the preferred axis of rotation.
6. The **divergence** of $\vec{F} =< P, Q, R >$ is
$$\text{div}\vec{F} = P_x + Q_y + R_z = \nabla \cdot \vec{F}.$$
7. Properties:

(a) $\text{div}(\text{curl}\vec{F}) = 0$ if P, Q, and R have continuous second-order partials.

(b) We say $\vec{F}$ is **incompressible** if $\text{div}\vec{F} = 0$.

(c) The divergence is a measure of how much the vector field wants to spread out away from a point.

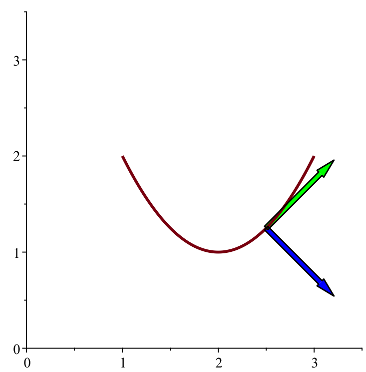
8. **Green's Theorem Revisited**:

$$\oint_C \vec{F} \cdot \vec{r} = \oint_C Pdx + Qdy = \iint_D (Q_x - P_y)dA = \iint_D (\text{curl}\vec{F}) \cdot \hat{k}dA.$$

9. This leads us to a new kind of path integral: instead of projecting $\vec{F}$ onto the tangent to the path we are traveling, we project it onto the normal:

$$\oint_C \vec{F} \cdot \hat{n}ds = \oint_C Pdy - Qdx = \iint_D (P_x + Q_y)dA = \iint_D \text{div}\vec{F}dA.$$

This is a **flux** integral, as it represents the flow through C.



10. Examples p. 1068: #2, 9-11, 15, 17, 19

Next Time

1. Watch 16.6 [∼43 minutes]