MATH 249

Today

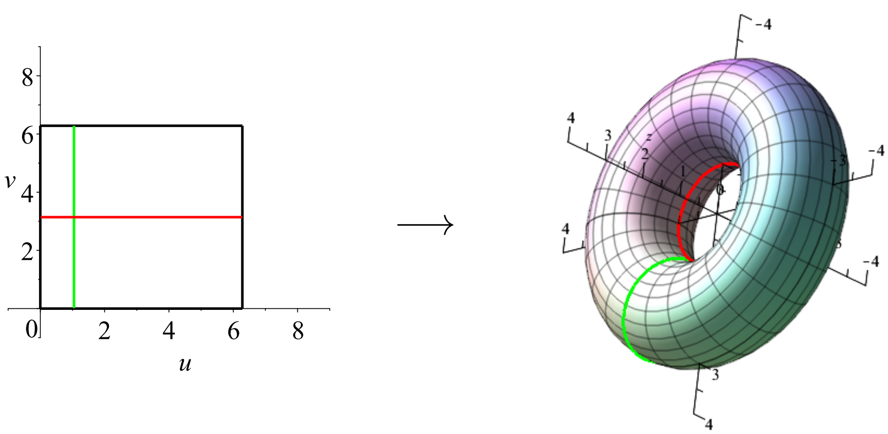
1. 16.6: Parametric Surfaces (Understand what a parametric surface is and be able to parametrize simple surfaces.)

2. WeBWorK

16.6: Parametric Surfaces

1. A **parametric surface** is a surface S for which x, y, and z depend on two parameters: $\vec{r}(u,v) = < x(u,v), y(u,v), z(u,v) >$ over some domain D in the uv-plane.

2. Holding u or v constant produces **grid curves** on the surface.



3. The normal to the tangent plane to a parametric surface is given by $\vec{r}_u \times \vec{r}_v$ (since both are tangent to the surface) provided they are defined and not parallel.

4. If $\vec{r}_u \times \vec{r}_v \neq \vec{0}$, then S is called **smooth**.

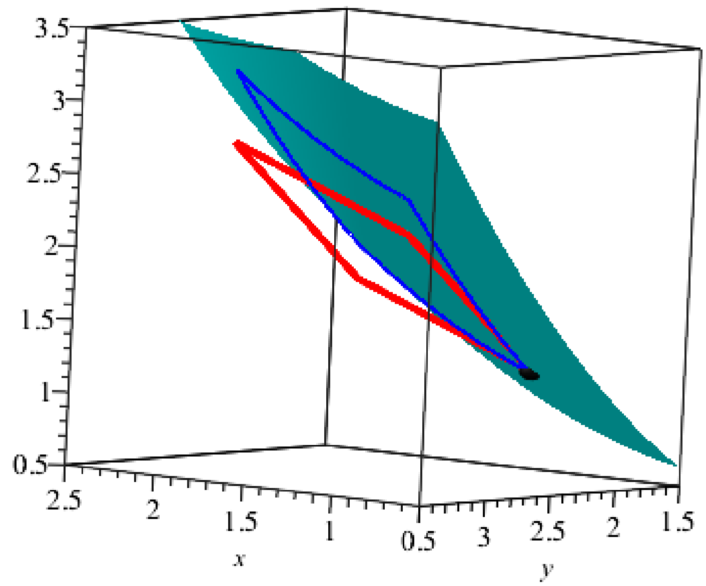
5. **Surface Area**: To find the surface area of S, we subdivide the domain D (in the uv-plane) in our usual way. The image of each subregion is a small patch on the surface S that can be approximated by a parallelogram with sides parallel to $\vec{r}_u$ and $\vec{r}_v$ and scaled by Δu and Δv, respectively. The area of that parallelogram is $|\vec{r}_u \times \vec{r}_v|\Delta A$, where $\Delta A = \Delta u \Delta v$. Thus, the surface area is given by

$$\lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} |\vec{r}_u(u_{ij}^*, v_{ij}^*) \times \vec{r}_v(u_{ij}^*, v_{ij}^*)|\Delta A,$$

which we can recognize as a double integral:

$$\iint_D |\vec{r}_u \times \vec{r}_v| dA,$$

where the integral is taken over the domain D in the uv-plane.



6. If $S : z = f(x, y)$, this becomes $\vec{r} =< x, y, f(x, y) >$, giving $\vec{r}_x =< 1, 0, f_x >$ and $\vec{r}_y =< 0, 1, f_y >$, and $|\vec{r}_x \times \vec{r}_y| = \sqrt{1 + f_x^2 + f_y^2}$. Thus the surface area in this case is given by

$$\iint_D \sqrt{1 + f_x^2 + f_y^2} dA.$$

7. Example: $\vec{r}(u, v) =< u \cos v, u \sin v, u >$ (See Maple file)

8. Examples p. 1078: #4, 35, 37, 47

Next Time

1. Watch 16.7 [~53 minutes]

2. Homefun