MATH 253

Today

1. 2.4 Subspaces continued (Understand what a subspace is and how to prove a subset of a vector space is a subspace. Recognize examples of subspaces.)
2. 2.5 Linear combinations and span (Understand definitions of linear combinations and span; express a vector as a linear combination of other vectors.)
3. WeBWorK

Where is today's material used?

1. Physics (solutions to Maxwell's equations in free space, quantum mechanics, Fourier series – everywhere we dealt with subspaces.)
2. Math (Linear combinations appear throughout mathematics)

TSST

Definition 0.1. Let V be a vector space over a field F. Then $W \subseteq V$ is a **subspace** of V if W is also a vector space over F using the same operations as V.

Theorem 1. *Let V be a vector space over a field F, and let W be a nonempty subset of V. Then W is a subspace of V if and only if W is closed under addition and scalar multiplication.*

Linear combinations and the span of a set of vectors

1. Let V be a vector space over F, and let $v_1, \dots, v_n \in V$. A **linear combination** of $v_1, \dots, v_n$ is a vector $v = a_1v_1 + \dots a_nv_n$ for some $a_1, \dots, a_n \in F$.
2. Let V be a vector space over F, and let $S \subseteq V$ be nonempty. The **span** of S is the set $\text{span}(S)$ of all finite linear combinations of vectors in D. We set $\text{span}(\emptyset) = \{0\}$.

Theorem 2. *Let V be a vector space.*

1. *If $S \subseteq V$, then span(S) is a subspace of V and $S \subseteq$ span(S).*
2. *If $S \subseteq S' \subseteq V$, then span$(S) \subseteq$ span(S').*
3. *$v \in$ span(S) if and only if span$(S \cup \{v\}) =$ span(S).*
4. *If W is a subspace of V and $S \subseteq W$, then span$(S) \subseteq W$.*

Definition 0.2. The **row space** of a matrix A is the span of the vectors formed by its rows, denoted $\mathbf{R}(A)$.

Theorem 3. *If $A_{m\times n}$ is row equivalent to $B_{m\times n}$, then $\mathbf{R}(A) = \mathbf{R}(B)$.*

Next Time

1. 2.5 Linear combinations and span
2. Note: 3 proofs due Wednesday (2.4).