MATH 253

Today

1. 2.7 Basis and dimension (Understand the definition of basis, the fact that every basis has the same size, and the roles of linear independence and spanning in this.)

2. WeBWorK

Where is today's material used?

1. Physics (solutions to Maxwell's equations in free space, quantum mechanics, Fourier series – everywhere we dealt with subspaces.)

2. Math (Bases appear throughout mathematics)

Theorem 1. *Let V be a vector space, and let $v, v_1, \ldots, v_n \in V$ and $S \subseteq V$.*

1. *(From 2.5) $v \in$span(S) if and only if span$(S \cup \{v\})$ =span(S).*

2. *(From 2.6) $v_1, \ldots, v_n$ are linearly dependent if and only if one lies in the span of the others.*

Theorem 2. *Let V be a vector space, and let $v, v_1, \ldots, v_n \in V$. If $v_1, \ldots, v_n$ span V and $u_1, \ldots, u_{n+1} \in V$, then $u_1, \ldots, u_{n+1} \in V$, are linearly dependent. (If n vectors span V, then any larger set of vectors is linearly dependent.)*

Corollary 0.1. *With notation as in the theorem, if $u_1, \ldots, u_m$ are linearly independent, then $m \leq n$.*

Theorem 3. *Let V be a vector space, and let $v_1, \ldots, v_n \in V$ be linearly independent. Then $v_1, \ldots, v_{n+1}$ are linearly independent if and only if $v_{n+1} \notin$span$(v_1, \ldots, v_n)$.*

Theorem 4. *Let $B_{m \times n}$ be a matrix in reduced row echelon form, and let $r, j_1, \ldots, j_r$ be the constants associated with B. Then the nonzero rows $B_1, \ldots, B_r$ of B are linearly independent.*

Next Time

1. 2.8 Rank