

# MATH 253

## Today

1. 2.7 Basis and dimension (Understand the definition of basis, the fact that every basis has the same size, and the roles of linear independence and spanning in this.)
2. 2.8 Rank (Understand the definitions of rank and nullity and the Rank-Nullity Theorem. Understand the connections to solving systems of equations.)
3. WeBWorK/Proofs

### Where is today's material used?

1. Math
2. Anywhere systems of equations are solved.

**Theorem 1.** *Let  $A_{m \times n}$  be a matrix over a field  $F$ . Then  $\dim(R(A)) = \dim(C(A))$ .*

*Proof.* Let  $B = RREF(A)$ .

1.  $\dim R(A) = \dim R(B)$ . ( $R(A) = R(B)$ .)
2.  $\dim R(B) = \dim C(B)$ . (The  $r$  nonzero rows of  $B$  are LI and span  $R(B)$ . The  $r$  pivot columns are LI and span  $C(B)$ .)
3.  $\dim C(B) = \dim C(A)$ .

□

## Next Time

1. 2.9
2. Note: 2 proofs due Monday (2.7).