MATH 253

Today

1. 5.6 The Cayley-Hamilton Theorem (Understand the Cayley-Hamilton Theorem and its applications. Understand the minimum polynomial of a matrix.)
2. WeBWorK

Let $A = \begin{bmatrix} 0.4 & 0.3 \\ -p & 1.2 \end{bmatrix}$. When $p = 0.325$, A has eigenvalues 1.05 and 0.55 with corresponding **linearly independent** eigenvectors $\begin{bmatrix} 0.46 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, respectively.

If the initial state vector is $v = \begin{bmatrix} O_0 \\ S_0 \end{bmatrix}$, then we can write v in the eigenvector basis as, say, $v = a_1 \begin{bmatrix} 0.46 \\ 1 \end{bmatrix} + a_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

After n iterations the state is $A^n v = a_1(1.05)^n \begin{bmatrix} 0.46 \\ 1 \end{bmatrix} + a_2 0.55^n \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, which tends to infinity for both populations.

The eigenvalues for $p = 0.5$ are 0.7 and 0.9, so a similar analysis gives both populations tending to 0.

If $p = 0.4$, then 1 is an eigenvalue, so the population levels off.

Next Time

1. Review
2. 2 proofs due Friday (5.5: 20; 5.6: 7)