

MATH 253

Today

1. Review

Review

1. Topics:

- (a) Vector spaces

- i. Definition
- ii. Operations
- iii. Properties
- iv. Examples

- (b) Subspaces (Two-Step Subspace Test)

- (c) Linear combinations

- (d) Span

- (e) Theorem 2.5.1 (especially (b) and (c))

- (f) Linear independence

- i. Definition
- ii. Equivalent conditions
- iii. Be ready to prove a set of vectors is linearly independent
- iv. Theorem 2.6.1 (v_1, \dots, v_n LD iff one is in the span of the others)
- v. Theorem 2.6.5 (If V is spanned by n vectors, any $n+1$ vectors are LD.)
- vi. Theorem 2.6.7 If v_1, \dots, v_n are LI, then v_1, \dots, v_n, v are LI iff $v \notin \text{span}(v_1, \dots, v_n)$.

- (g) Dimension and basis

- i. Definitions
- ii. Theorem 2.7.1 (Dimension is well defined)
- iii. Theorem 2.7.6 (Extension Theorem)
- iv. If $\dim V = n$, then $v_1, \dots, v_n \in V$ are linearly independent if and only if they span V .

- v. Uniqueness of representation in a basis
- (h) Rank and the Rank-Nullity Theorem (Theorem 2.9.1: if A is $m \times n$, then $\text{rank}(A) + \text{nullity}(A) = n$.)
- (i) Row and column spaces: Let A be a matrix and $B = RREF(A)$.
 - i. Definitions
 - ii. $\mathbf{R}(A) = \mathbf{R}(B)$
 - iii. $\dim \mathbf{R}(A) = \dim \mathbf{R}(B)$ ($\text{rank}(A) = \text{rank}(B)$.)
 - iv. $\dim \mathbf{R}(A) = \dim \mathbf{C}(A)$ ($\text{rank}(A) = \dim \mathbf{R}(A) = \dim \mathbf{C}(A)$.)
 - v. Theorem 2.8.2 ($\text{rank}(A)$ is the number of nonzero rows in B .)

2. Logistics:

- (a) We can start 5 minutes early and run 5 minutes late (7:55-9:05).
- (b) The exam is intended to be a 50-minute exam.
- (c) There will be one or two short proofs. The rest will be computational or conceptual.
- (d) You may use Python or a calculator, but I will ask you explicitly to perform some computations by hand.
- (e) You may load our exam template (I will share it with you), and you may use our Python info sheet.
- (f) The problems will be on one page. I will provide blank paper for you to work on. Please leave the upper left corner blank (for a staple) and write on only one side of the page.

Questions

Next Time

1. Exam