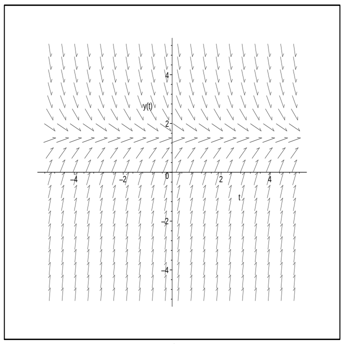
Solutions to Homework Assignment 1

MATH 256-01
Section 1.1, Page 8
To Keep: 1, 2, 3, 7, 9, 13, 14, 15, 16, 17, 20, 23

1. All solutions converge to the stable solution $y = 1.5$.

2. All solutions (except $y = 1.5$) diverge from the solution $y = 1.5$.

3. All solutions (except $y = -1.5$) diverge from the solution $y = -1.5$.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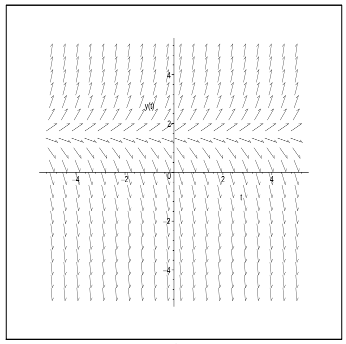
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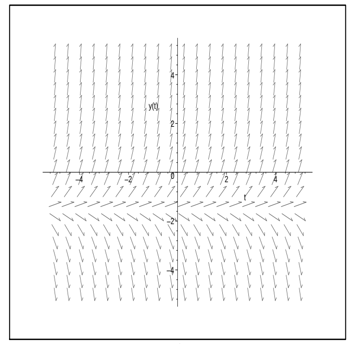
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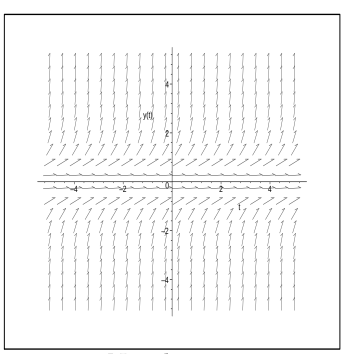
7. We need a horizontal tangent line for all points at which $y = 3$; that is, we need $\dfrac{dy}{dt} = 0$ whenever $y = 3$. We know that $\dfrac{dy}{dt} = ay + b$, so we need $3a + b = 0$. In addition, we need for other solutions to approach 3 as $t \to \infty$. Thus, if $y > 3$, we need $\dfrac{dy}{dt} < 0$, and if $y < 3$, we need $\dfrac{dy}{dt} > 0$. With $b = -3a$, we have $\dfrac{dy}{dt} = ay - 3a = a(y-3)$, so to make this negative for $y > 3$ and positive for $y < 3$, we require only that $a < 0$. Putting $a = -1$ gives $\dfrac{dy}{dt} = 3 - y$, which satisfies all of our conditions. (Any negative choice of a would work as well.)

9. Reasoning as in Exercise 7, we see that we need $b = -2a$ and $a > 0$. The equation $\dfrac{dy}{dt} = y - 2$ will satisfy the given conditions.

13. The stable solution is $y = 0$. For other solutions, if $y > 0$, then the solution diverges from equilibrium; if $y < 0$, then the solution converges to equilibrium.

14. Here, $y = 0$ and $y = 2$ are both equilibrium solutions. All solutions except $y = 0$ diverge from $y = 0$. If $y > 2$, then the corresponding solution diverges from $y = 2$; if $0 < y < 2$, then the solution converges to $y = 2$.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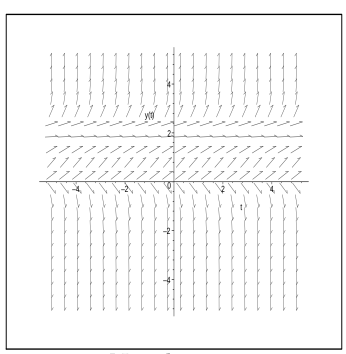
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15. Let $A(t)$ represent the total amount of the chemical in the pond t minutes after we begin.

    (a) We are given the following information on the rate of change of A with respect to time: the pond is gaining $(0.01)(300) = 3$ grams per minute of the chemical, and at the same time it is losing $A(t) \cdot \frac{300}{1000000} = 0.0003A(t)$ grams per minute. Thus, the net rate of change in the amount of chemical in the pond is
    $$A'(t) = 3 - 0.0003A(t).$$

    (b) Comparing this with Exercise 7, we see that the solutions will all converge to 10000 grams. In a 1,000,000 gallon pond, this gives a concentration of 0.01 grams per gallon, which is reasonable since that is the concentration in the solution flushing the pond.

16. The surface area of a spherical raindrop of radius r is $4\pi r^2$. Since no water is being added to the drop, we have a net rate of change of $\frac{dV}{dt} = -4\pi r^2 \cdot k$, where k is the (positive) constant of proportionality. However, we now have three variables: V, r, and t; we need to simplify this by eliminating r. Since the volume of our raindrop is $V = \frac{4}{3}\pi r^3$, we have $r = \left(\frac{3}{4\pi}V\right)^{1/3}$.

    We arrive at $\frac{dV}{dt} = -4k\pi\left(\frac{3}{4\pi}V\right)^{2/3}$. Absorbing all of the constants into the single constant C gives $\frac{dV}{dt} = -CV^{2/3}$, which is a little easier to read.

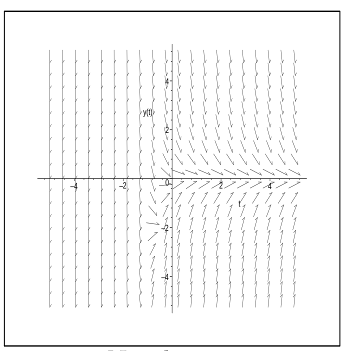
17. Let $A(t)$ be the amount of the drug (in mg) in the patient's bloodstream t hours after we begin.

    (a) The bloodstream gains 500 mg/hr, but loses $0.4A(t)$ mg/hr. Thus, the rate of change in the amount of the drug present in the bloodstream at time t is
    $$A'(t) = 500 - 0.4A.$$

    (b) Over time, the quantity of the drug in the patient's bloodstream stabilizes at 1250 mg. (This is the quantity at which $A'(t)$ is zero.)

20. The solutions all approach 0 as $t \to \infty$.

23. There appears to be an oscillating solution that all other solutions diverge from. The divergent solutions approach $\pm\infty$.



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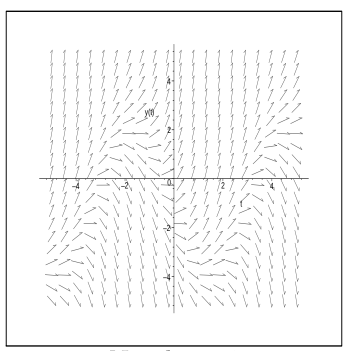

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