Solutions to Homework Assignment 3

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To Keep: 1-21, 23, 28

1. Second order linear.
2. Second order linear.
3. Fourth order linear.
4. First order nonlinear.
5. Second order nonlinear.
6. Third order linear.
7. Since $y_1'' = e^t = y_1$, e^t is a solution of $y'' - y = 0$. Likewise, $y_2'' = cosht$. (Recall that $\frac{d}{dt}\cosh t = sinht$ and $\frac{d}{dt}\sinh t = \cosh t$.)
8. $y_1' = -3e^{-3t}$, and $y_1'' = 9e^{-3t}$. Thus $y_1'' + 2y' - 3y = 9e^{-3t} - 6e^{-3t} - 3e^{-3t} = 0$.
   $y_2' = y_2'' = e^t$, so $y_2'' + 2y_2' - 3y_2 = e^t + 2e^t - 3e^t = 0$.
9. $y' = 3 + 2t$, so $ty' - y = t(3 + 2t) - (3t + t^2) = 3t + 2t^2 - 3t - t^2 - t^2$, as desired.
10. $y_1' = 1/3, y_1'' = y_1''' = y_1'''' = 0$. Thus $y_1'''' + 3y_1''' + 3y = 0 + 0 + t = t$.
   $y_2' = -e^{-t} + 1/3, y_2'' = e^{-t} = -y_2''' = y_2''''$. Thus $y_2'''' + 4y_2''' + 3y = e^{-t} - 4e^{-t} + 3e^{-t} + t = t$.
11. $y_1' = \frac{1}{2}t^{-1/2}$ and $y_1'' = -\frac{1}{4}t^{-3/2}$. Thus $2t^2y_1'' + 3ty_1' - y_1 = -2t^2(t^{-3/2}/4) + 3t(t^{-1/2}/2) - t^{1/2} = -t^{1/2}/2 + 3t^{1/2}/2 - t^{1/2} = 0$.
   $y_2' = -t^{-2}, y_2'' = 2t^{-3}$. Thus $2t^2y_2'' + 3ty_2' - y_2 = 4t^{-1} - 3t^{-1} - t^{-1} = 0$.
12. $y_1' = -2t^{-3}, y_1'' = 6t^{-4}$. Now $t^2y_1'' + 5ty_1' + 4y_1 = 6t^{-2} - 10t^{-2} + 4t^{-2} = 0$.
   $y_2' = -2t^{-3}\ln t + t^{-3}, y_2'' = 6t^{-4}\ln t - 2t^{-4} - 3t^{-4} = 6t^{-4}\ln t - 5t^{-4}$. Thus $t^2y_2'' + 5ty_2' + 4y_2 = (6t^{-2}\ln t - 5t^{-2}) + (-10t^{-2}\ln t + 5t^{-2}) + (4t^{-2}\ln t) = 0$.
13. $y' = (-\sin t)\ln\cos t - (\cos t)\cdot\frac{\sin t}{\cos t} + \sin t + t\cos t = -\sin t\ln\cos t + t\cos t$, and $y'' = -\cos t\ln\cos t + \frac{\sin^2 t}{\cos t} + \cos t - t\sin t = -\cos t\ln\cos t + \frac{1}{\cos t} - t\sin t$. Thus,
   $y'' + y = -\cos t\ln\cos t + \frac{1}{\cos t} - t\sin t + \cos t\ln\cos t + t\sin t = \frac{1}{\cos t} = \sec t$.
14. $y' = 2te^{t^2}\int_0^t e^{-s^2}ds + e^{t^2}\cdot e^{-t^2} + 2te^{t^2} = 2te^{t^2}\int_0^t e^{-s^2}ds + 1 + 2te^{t^2}$. Thus $y' - 2ty = 2te^{t^2}\int_0^t e^{-s^2}ds + 1 + 2te^{t^2} - 2te^{t^2}\int_0^t e^{-s^2}ds - 2te^{t^2} = 1$.
15. Let $y = e^{rt}$. Then $y' + 2y = re^{rt} + 2e^{rt} = 0$ if and only if $r = -2$.
16. Let $y = e^{rt}$. Then $y'' - y = r^2e^{rt} = e^{rt} = 0$ if and only if $r = \pm 1$.
17. Let $y = e^{rt}$. Then $y'' + y' - 6y = r^2e^{rt} + re^{rt} - 6e^{rt} = e^{rt}(r^2 + r - 6) = e^{rt}(r + 3)(r - 2) = 0$ if and only if $r = -3$ or $r = 2$.
18. Let $y = e^{rt}$. Then $y''' - 3y'' + 2y' = r^3e^{rt} - 3r^2e^{rt} + 2re^{rt} = re^{rt}(r - 2)(r - 1) = 0$ if and only if $r \in \{0, 1, 2\}$.

19. Let $y = t^r$. Then $t^2y'' + 4ty' + 2y = r(r-1)t^r + 4rt^r + 2t^r = t^r(r^2 + 3r + 2) = t^r(r+1)(r+2) = 0$ if and only if $r = -1$ or $r = -2$.

20. Let $y = t^r$. Then $t^2y'' - 4ty' + 4y = r(r-1)t^r - 4rt^r + 4t^r = t^r(r^2 - 5r + 4) = t^r(r-1)(r-4) = 0$ if and only if $r = 1$ or $r = 4$.

21. This is a linear second-order PDE.

23. This is a linear fourth-order PDE.

28. $u_x = -\dfrac{x}{2\alpha^2 t}(\pi/t)^{1/2}e^{-x^2/(4\alpha^2 t)}$, and $u_{xx} = -\dfrac{1}{2\alpha^2 t}(\pi/t)^{1/2}e^{-x^2/(4\alpha^2 t)} + (\pi/t)^{1/2}\dfrac{x^2}{4\alpha^4 t^2}e^{-x^2/(4\alpha^2 t)}$.

$u_t = -\dfrac{\pi}{2t}(\pi/t)^{-1/2}e^{-x^2/(4\alpha^2 t)} + (\pi/t)^{1/2}\dfrac{x^2}{4\alpha^2 t^2}e^{-x^2/(4\alpha^2 t)}$