Solutions to Homework Assignment 6

MATH 256-01
Section 2.3, Page 57
Problems: 1, 3, 4, 8, 9, 15, 21

1. Let $D(t)$ represent the amount of dye t minutes after we start rinsing. We are told that $D(0) = 200$. Our rate of change is $D'(t) = -\frac{2}{200}D(t)$. MAPLE says the solution to this initial value problem is $D(t) = 200e^{-t/100}$. This reaches 1% of its original value when $200e^{-t/100} = 2$, or $e^{-t/100} = 0.01$. Thus $-t/100 = \ln(0.01)$, so $t = -100\ln(0.01) \approx 460.5$ minutes.

3. We are gaining 1 pound of salt per minute and losing 2/100 of the salt in the tank per minute. Thus if $S(t)$ represents the amount of salt in the tank at time t, we have $S'(t) = 1 - 0.02S(t)$ for $t \leq 10$ and $S(0) = 0$. MAPLE gives $S(t) = 50 - 50e^{-t/50}$. At $t = 10$, this is $S(10) \approx 9.0635$.

   After 10 minutes, we get $S'(t) = -0.02S(t)$, with $S(10) \approx 9.0635$. MAPLE gives $S(t) \approx 11.07e^{-0.02t}$. Now $S(20) \approx 7.42$ pounds.

4. We have $S(0) = 100$ and the net amount of water in the tank at time t is $200 + 3t - 2t = 200 + t$ gallons. Thus $S'(t) = 3 - \frac{2}{200+t}S(t)$. MAPLE gives $S(t) = 200 + t - \frac{4000000}{(200+t)^2}$.

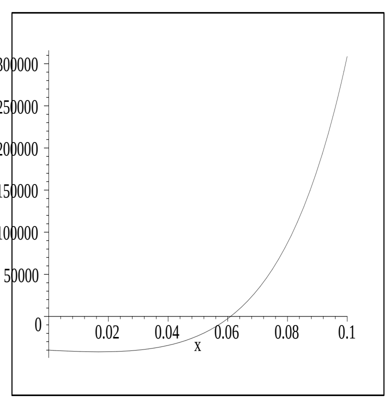
   At $t = 300$ minutes, the tank is on the point of overflowing. At this point, the amount of salt in the tank is 484 pounds, for a concentration of 0.968 pounds per gallon. That's getting fairly close to 1 pound per gallon, which is what we expect in the limit.

8. We will use the results of Example 2 of the section: $S(t) = S_0e^{rt} + (k/r)(e^{rt} - 1)$.

   (a) Person A: We need the balance after 10 years, first: $S_A(10) = 0e^{0.08(10)} + (2000/0.08)(e^{0.08(10)} - 1) \approx 30638.52$. Now we start over with a new S_A having $k = 0$ and $S_0 = 30368.52$. $S_A(30) = 30368.52e^{0.08(30)} \approx 337733.81$.
   Person B: $S_B(t) = (2000/0.08)(e^{0.08(30)} - 1) \approx 250579.41$. Wow!

   (b) $S_A(t) = (2000/r)(e^{r(10)} - 1)$ for the first 10 years for person A. This then becomes the starting investment for person A's next 30 years.
   $S_A(r) = (2000/r)(e^{10r} - 1)e^{30r}$.
   $S_B(r) = (2000/r)(e^{30r} - 1)$.

   (c) Below is a graph of $S_A(r) - S_B(r)$.



   (d) They are equal at about $r = 0.061$, or 6.1%.

9. Again using the results of Example 2, we have $S(t) = 8000e^{0.1t} - (k/0.1)(e^{0.1t} - 1)$. In order to have $S(3) = 0$, we must have

$$0 = 8000e^{0.1(3)} - (k/0.1)(e^{0.1(3)} - 1)$$
$$10k(0.3499) = 10798.87$$
$$k = 3086.64.$$

Thus, in three years the borrower pays \$9259.91, of which \$1259.91 is interest.

15. Let $M(t)$ denote the number of mosquitoes after t days. We are told that $M(0) = 200000$ and $M'(t) = kM(t) - 20000$. MAPLE solves this as $M(t) = \dfrac{20000 + 20000e^{kt}(10k - 1)}{k}$. Also, if there were no predators, we would have $M'(t) = kM(t)$, so $M(t)$ would be $200000e^{kt}$. For this to double in 7 days, we would have to have $400000 = 200000e^{7k}$, or $e^{7k} = 2$, giving $k = \dfrac{\ln(2)}{7} \approx 0.099$. Therefore we have (for the actual $M(t)$)

$$M(t) = 201977.306 - 1977.306e^{0.099t}.$$

(Note: My t is in days; the book's t is in weeks.)

21. Recall from Chapter 1 that $m\dfrac{dv}{dt} = mg$ if we neglect air resistance. Thus, $v' = g$, so $v = gt + C$. Since $v(0) = 20$, we must have $C = 20$.

Now $v = x'$, so $x' = gt + 20$. Therefore, $x = \dfrac{1}{2}gt^2 + 20t + C$. Since the building is 30 meters tall, we have $C = 30$. This gives $x(t) = -4.9t^2 + 20t + 30$ (using $g = -9.8\text{m/s}^2$).

(a) This happens when $v = 0$, which occurs at $t = 20/g = 2.041$ seconds. At this time, the height is 50.41 meters.

(b) It hits when $x = 0$, which is about 5.25 seconds (according to MAPLE).

(c) The line is the velocity.