Autonomous Equations Homework
MATH 256

Problems from Boyce and DiPrima, 7th Edition

For 1-6, sketch $f(y)$ versus y, determine the equilibrium points, and classify each as stable or unstable.

1. $\dfrac{dy}{dt} = ay + by^2,\ a, b > 0, y_0 \geq 0$

2. $\dfrac{dy}{dt} = ay + by^2, a, b > 0,\ -\infty < y_0 < \infty$

3. $\dfrac{dy}{dt} = y(y-1)(y-2), y_0 \geq 0$

4. $\dfrac{dy}{dt} = e^y - 1, -\infty < y_0 < \infty$

5. $\dfrac{dy}{dt} = e^{-y} - 1, -\infty < y_0 < \infty$

6. $\dfrac{dy}{dt} = -\dfrac{2\arctan y}{1+y^2}, -\infty < y_0 < \infty$

7. An equilibrium solution is **semistable** if solutions on one side approach it and solutions on the other side move away from it. Show that $y(t) = 1$ is a semistable solution of $\dfrac{dy}{dt} = k(1-y)^2$, where $k > 0$. Then solve the IVT with $y(0) = y_0$.

8. $\dfrac{dy}{dt} = -k(y-1)^2,\ k > 0, -\infty < y_0 < \infty$. Determine the equilibrium solutions and classify each as stable, unstable, or semistable.

9. $\dfrac{dy}{dt} = y^2(y^2-1), -\infty < y_0 < \infty$. Determine the equilibrium solutions and classify each as stable, unstable, or semistable.

15. Suppose that a population $y(t)$ obeys the equation $\dfrac{dy}{dt} = ry(1 - y/K)$.

    (a) If $y_0 = K/3$, at what time τ has the population doubled?

    (b) If $y_0/K = \alpha$, at what time T do we have $y(T)/K = beta$, where $0 < \alpha, \beta < 1$? What happens to T as $\alpha \to 0? \beta \to 1$?

26. A certain chemical reaction producing chemical X with concentration $x(t)$ can be modeled with the equation $\dfrac{dx}{dt} = \alpha(p-x)(q-x)$, where p and q are the initial concentrations of the chemicals producing X, where $\alpha > 0$. Assume $x(0) = 0$ and solve the IVT.