Solutions to Homework Assignment 16

MATH 256-01
Section 4.2, Page 219
Problems: 1-10, 11-23 odd, 29-35 odd

1. $1+i$ has magnitude $\sqrt{2}$ and polar angle $\pi/4$, so we have $1+i=\sqrt{2}e^{i\pi/4+2in\pi}$.
2. $-1+\sqrt{3}i$ has magnitude 2 ($(-1)^2+(\sqrt{3})^2=4$). Thus $2\cos\theta=-1$ and $2\sin\theta=\sqrt{3}$, so $\theta=2\pi/3+2n\pi$. Thus $-1+\sqrt{3}i=2e^{2i\pi/3+2in\pi}$.
3. This is a real number, so its polar angle is $0+2n\pi$; we have $-3=-3e^{0+2in\pi}$.
4. $-i$ has polar angle $-\pi/2$ and magnitude 1, so $-i=e^{-i\pi/2+2in\pi}$.
5. We have magnitude 2, and $2\cos\theta=\sqrt{3}, 2\sin\theta=-1$, so $\theta=-\pi/6+2n\pi$. Thus $\sqrt{3}-i=2e^{-i\pi/6+2in\pi}$.
6. The polar angle is $5\pi/4$ and the magnitude is $\sqrt{2}$, so $-1-i=\sqrt{2}e^{5i\pi/4+2in\pi}$.
7. We have $1=e^{0+2n\pi i}$, so $1^{1/3}=e^{2n\pi i/3}=\cos(2\pi/3)+i\sin(2\pi/3)=-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}$ (if $n=1$) or $\cos(4\pi/3)+i\sin(4\pi/3)=-\dfrac{1}{2}-i\dfrac{\sqrt{3}}{2}$ (if $n=2$).
8. $1-i=\sqrt{2}e^{-i\pi/4+2in\pi}$, so $(1-i)^{1/2}=\sqrt[4]{2}e^{-i\pi/8+in\pi}$, which is either $\sqrt[4]{2}e^{-i\pi/8}$ or $\sqrt[4]{2}e^{7i\pi/8}$.
9. $1^{1/4}=e^{i\pi/2+2in\pi}$, or one of $1, i, -1, -i$.
10. We want $\sqrt{2}(e^{i\pi/3}+2in\pi)^{1/2}=\sqrt{2}e^{i\pi/6+in\pi}$, which is $\pm\sqrt{2}(\cos(\pi/6)+i\sin(\pi/6))=\pm\sqrt{2}(\sqrt{3}/2+i/2)$.
11. The characteristic equation is $r^3-r^2-r+1=0$, or $(r-1)^2(r+1)=0$. The general solution is therefore $y(t)=c_1e^t+c_2te^t+c_3e^{-t}$.
13. The characteristic equation is $2r^3-4r^2-2r+4=0$, or $2(r-2)(r-1)(r+1)=0$, so the general solution is $y(t)=c_1e^{2t}+c_2e^t+c_3e^{-t}$.
15. The characteristic equation is $r^6+1=0$, so $r=(-1)^{1/6}$, or $e^{i\pi/6+in\pi/3}$ for $n=0,1,2,3,4,5$. $\cos(\pi/6)=\sqrt{3}/2$ and $\sin(\pi/6)=1/2$. Our roots are $\sqrt{3}2+i/2, 0+i, -\sqrt{3}/2+i/2, -\sqrt{3}/2-i/2, 0-i$, and $\sqrt{3/2}-i/2$. (Notice that we do get conjugate pairs.) The corresponding solution is $y(t)=c_1\cos t+c_2\sin t+c_2e^{\sqrt{3}t/2}(c_3\cos(t/2)+c_4\sin(t/2))+e^{-\sqrt{3}t/2}(c_5\cos(t/2)+c_6\sin(t/2))$. Ugh!
17. The characteristic equation is $r^6-3r^4+3r^2-1=0$, or $(r^2-1)^3=0$. We have thrice repeated roots ±1, so the general solution is $y(t)=c_1e^t+c_2te^t+c_3t^2e^t+c_4e^{-t}+c_5te^{-t}+c_6t^2e^{-t}$.
19. The characteristic equation is $r^5-3r^4+3r^3-3r^2+2r=0$, or $r(r^4-3r^3+3r^2-3r+2)=0$. It is not hard to see that $r=1$ is a root, leaving $r(r-1)(r^3-2r^2+r-2)=0$. It is also not hard to see that $r=2$ is a root of this, so we have $r(r-1)(r-2)(r^2+1)=0$. The roots are $r=0,1,2,\pm i$, so the general solution is $c_1+c_2e^t+c_3e^{2t}+c_4\cos t+c_5\sin t$.
21. The characteristic equation is $r^8+8r^4+16=0$, or $(r^4+4)^2=0$. We have four repeated roots, the solutions of $r^4=-4$. These are $\sqrt{2}e^{-\pi/4+in\pi/2}$, $n=0,1,2,3$. We have $\sqrt{2}(\sqrt{2}/2-i\sqrt{2}/2)=1-i, \sqrt{2}(\sqrt{2}/2+i\sqrt{2}/2)=1+i, \sqrt{2}(-\sqrt{2}/2+i\sqrt{2}/2)=-1+i$, and $\sqrt{2}(\sqrt{2}/2-i\sqrt{2}/2)=1-i$. The general solution is $y(t)=e^t(c_1\cos t+c_2\sin t)+te^t(c_3\cos t+c_4\sin t)+e^{-t}(c_5\cos t+c_6\sin t)+te^{-t}(c_7\cos t+c_8\sin t)$.
23. The characteristic equation is $r^3-5r^2+3r+1=0$. It's easy to see that $r=1$ is a solution, so we have $(r-1)(r^2-4r-1)=0$. The other solutions are $r=\dfrac{4\pm\sqrt{20}}{2}=2\pm\sqrt{5}$. The general solution is thus $c_1e^t+c_2e^{(2+\sqrt{5})t}+c_3e^{(2-\sqrt{5})t}$.

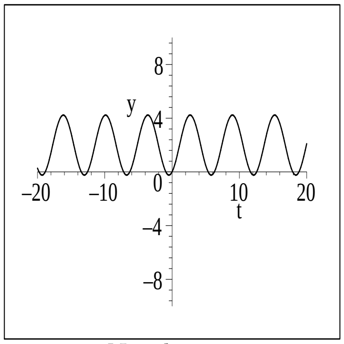
For 29-35 odd, I will use MAPLE to solve for the coefficients. I recommend you do likewise (with MAPLE or your graphing calculator).

29. $r^3 + r = 0$, so $r = 0, \pm i$. The general solution is $y(t) = c_1 + c_2 \cos t + c_3 \sin t$. I'll do this one by hand: $y(0) = c_1 + c_2 = 0, y'(0) = c_3 = 1$, and $y''(0) = -c_2 = 2$, so $c_1 = 2$. Thus $y(t) = 2 - 2\cos t + \sin t$. The graph is below; the solution oscillates as $t \to \infty$.

31. $r^4 - 4r^3 + 4r^2 = 0$, so $r^2(r-2)^2 = 0$. We have $y(t) = c_1 + c_2 t + c_3 e^{2t} + c_4 t e^{2t}$. MAPLE gives $y(t) = 2t - 3$. I'm not graphing that for you.

33. $2r^4 - r^3 - 9r^2 + 4r + 4 = 0$. $r = 1$ is a solution, leaving $(r-1)(2r^3 + r^2 - 8r - 4) = 0$. $r = 2$ is a zero of this, so we now have $(r-1)(r-2)(2r^2 + 5r + 2) = 0$, which factors as $(r-1)(r-2)(2r+1)(r+2) = 0$. The general solution is $y(t) = c_1 e^t + c_2 e^{2t} + c_3 e^{-t/2} + c_4 e^{-2t}$. MAPLE gives $y(t) = -\frac{2}{3}e^t - \frac{1}{10}e^{2t} - \frac{16}{15}e^{-t/2} - \frac{1}{6}e^{-2t}$. The graph is below; $y \to -\infty$ as $t \to \infty$.

35. $6r^3 + 5r^2 + r = 0$, so $r(3r+1)(2r+1) = 0$. The general solution is $y(t) = c_1 + c_2 e^{-t/3} + c_3 e^{-t/2}$. MAPLE gives $y(t) = 8 - 18e^{-t/3} + 8e^{-t/2}$. This approaches 8 as $t \to \infty$.



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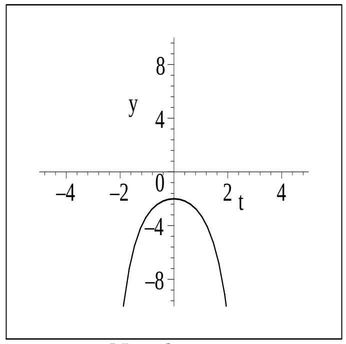

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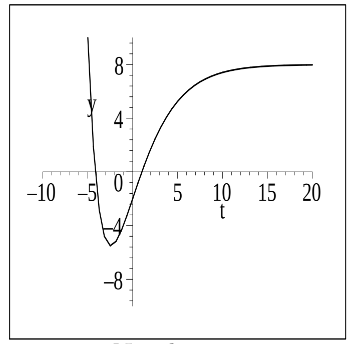

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