Solutions to Homework Assignment 19

MATH 256-01
Section 4.3, Page 224
Problems: 1-17 odd

1. The annihilator we need is $D(D+1)$, and the characteristic equation of the corresponding homogeneous equation is $r^3 - r^2 - r + 1 = 0$. The right-hand side factors as $r^2(r-1) - 1(r-1) = (r-1)^2(r+1)$. Therefore, our equation becomes $D(D+1)^2(D-1)^2y = 0$. The general solution of this is $c_1 + c_2e^{-t} + c_3te^{-t} + c4e^t + c_5te^t$. A particular solution has the form $Y(t) = A + Bte^{-t}$ (since the other terms solve the corresponding homogeneous equation).

   $Y'(t) = B(1-t)e^{-t}$, $Y''(t) = B(t-2)e^{-t}$, and $Y'''(t) = B(3-t)e^{-t}$. Thus we have $Be^{-t}[3 - t - (t - 2) - (1-t) + t] + A = 2e^{-t} + 3$. This gives $A = 3$ and $4B = 2$, so $B = 1/2$. Our particular solution is $Y(t) = 3 + \dfrac{1}{2}te^{-t}$, and the general solution is $3 + \dfrac{1}{2}te^{-t} + c_1e^t + c_2te^t + c_3e^{-t}$.

3. We need the annihilator $D^2(D+1)$; the equation becomes $D^2(D+1)^2(D^2+1)y = 0$. The general solution of this is $y(t) = c_1 + c_2t + c_3e^{-t} + c_4te^{-t} + c_5\cos t + c_6\sin t$. Our particular solution is $Y(t) = A + Bt + Cte^{-t}$; the other terms all satisfy the corresponding homogeneous equation.

   $Y'(t) = B + C(1-t)e^{-t}, Y''(t) = C(t-2)e^{-t}$, and $Y'''(t) = C(3-t)e^{-t}$. We get

   $Ce^{-t}[3 - t + t - 2 + 1 - t + t] + B + (A + Bt) = e^{-t} + 4t$. This gives $B = 4, A = -4$, and $2C = 1$, so $C = 1/2$. Thus $Y(t) = -4 + 4t + \dfrac{1}{2}te^{-t}$ and $y(t) = -4 + 4t + \dfrac{1}{2}te^{-t} + c_1e^{-t} + c_2\cos t + c_3\sin t$.

5. We need the annihilator $D^3(D-1)$, so our equation becomes $D^5(D-1)(D-2)(D+2)y = 0$. The general solution is $y(t) = c_1 + c_2t + c_3t^2 + c_4t^3 + c_5t^4 + c_6e^t + c_7e^{2t} + c_8e^{-2t}$. Our particular solution will be $Y(t) = At^2 + Bt^3 + Ct^4 + De^t$. This gives $Y'(t) = 2At + 3Bt^2 + 4Ct^3 + De^t$, $Y''(t) = 2A + 6Bt + 12Ct^2 + De^t, Y''' = 6B + 24Ct + De^t$, and $Y^{(4)} = 24C + De^t$. We have

   $(24C + De^t) - 4(2A + 6Bt + 12Ct^2 + De^t) = t^2 + e^t$, so $-3D = 1$, $-48C = 1, -24B = 0, 24C - 8A = 0$, so $B = 0, C = -1/48, A = 3C = -1/16$, and $D = -1/3$. Therefore $Y(t) = -\dfrac{1}{48}t^4 - \dfrac{1}{3}e^t$. The general solution is $y(t) = c_1 + c_2t + c_3e^{2t} + c_4e^{-2t} - \dfrac{1}{16}t^2 - \dfrac{1}{48}t^4 - \dfrac{1}{3}e^t$.

7. The annihilator we need is D^2, so the equation becomes $D^5(D^3+1)y = 0$, or $D^5(D+1)(D^2-D+1)y = 0$. We get from this a particular solution $Y(t) = At^3 + Bt^4$. $Y''' = 6A + 24Bt$ and $Y^{(6)}(t) = 0$, so we have $6A + 24Bt = t$. Thus $B = 1/24$ and $A = 0$. The general solution is $y(t) = c_1 + c_2t + c_3t^2 + c_4e^{-t} + c_5e^{t/2}\cos(\sqrt{3}t/2) + c_6e^{t/2}\sin(\sqrt{3}t/2) + \dfrac{1}{24}t^4$.

9. We have $D^3(D^2+4)y = 0$. Our particular solution is $Y(t) = At + Bt^2$, so $Y'(t) = A + 2Bt, Y''(t) = 2B$, and $Y'''(t) = 0$. Thus $4(A+2Bt) = t$, so $A = 0$ and $B = 1/8$. This gives $Y(t) = \dfrac{1}{8}t^2$. The general solution is $y(t) = c_1 + c_2\cos 2t + c_3\sin 2t + \dfrac{1}{8}t^2$. Since $y(0) = 0$, $c_1 + c_2 = 0$. Since $y'(0) = 0, 2c_3\cos 2t = 0$ (and hence $c_3 = 0$). $y''(0) = -4c_2 + \dfrac{1}{4} = 1$, so $c_2 = -3/16$, making $c_1 = 3/16$. Therefore, $y(t) = \dfrac{3}{16} - \dfrac{3}{16}\cos 2t + \dfrac{1}{8}t^2$.

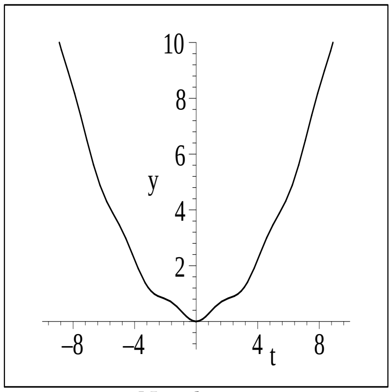
11. The annihilator $D^2(D-1)$ reproduces D and $D-1$, so the particular solution has the form $Y(t) = At + Bt^2 + Cte^t$. $Y'(t) = A + 2Bt + C(t+1)e^t, Y''(t) = 2B + C(t+2)e^t$, and $Y'''(t) = C(t+3)e^t$. We get

$$C(t+3)e^t - 3(2B + C(t+2)e^t) + 2(A + 2Bt + C(t+1)e^t) = t + e^t,$$

   so $-C = 1, -6B + 2A = 0$, and $4B = 1$. These give $B = 1/4, A = 3/4$, and $C = -1$, so $Y(t) = \dfrac{3}{4}t + \dfrac{1}{4}t^2 - te^t$. The general solution has the form

$$y(t) = c_1 + c_2e^t + c_3e^{2t} + \frac{3}{4}t + \frac{1}{4}t^2 - te^t.$$

Since $y(0) = 1$, $c_1 + c_2 + c_3 = 1$. Since $y'(0) = -\frac{1}{4}$, $c_2 + 2c_3 + \frac{3}{4} - 1 = -\frac{1}{4}$. Since $y''(0) = -\frac{3}{2}$, $c_2 + 4c_3 + \frac{1}{2} - 2 = -\frac{3}{2}$. This means $c_2 = -2c_3$ and $c_2 = -4c_3$, so $c_2 = c_3 = 0$. Thus $c_1 = 1$, and $y(t) = 1 + \frac{3}{4}t + \frac{1}{4}t^2 - te^t$.



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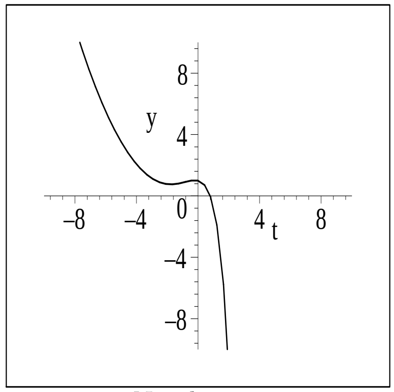

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13. The annihilator we need is $D^4(D-1)$, so the equation becomes $D^5(D-1)^3y = 0$. A particular solution is $Y(t) = At + Bt^2 + Ct^3 + Dt^4 + Et^2e^t$.

15. We need $(D-1)(D^2+1)$ for the annihilator, so we get $(D-1)(D^2+1)(D^2-1)^2y = 0$, or $(D-1)^3(D+1)^2(D^2+1)y = 0$. A particular solution is $Y(t) = At^2e^t + B\cos t + C\sin t$.

17. We need $D^3(D^2+1)^2$ to annihilate the right-hand side, so we get $D^4(D^2+1)^2(D-1)^2(D+1)$. Thus $Y(t) = At + Bt^2 + Ct^3 + D\cos t + C\sin t + Et\cos t + Ft\sin t$. (The only solution to the homogeneous solution that we reproduced was the constant solution.)