

Series Solutions near an Regular Singular Point

MATH 256

Problems from Boyce and DiPrima, 7th Edition (Section 5.7)

For 1-11, find the regular singular points, the indicial equation, and the exponents at the singularity.

1. $xy'' + 2xy' + 6e^x y = 0$

3. $x(x - 1)y'' + 6x^2 y' + 3y = 0$

5. $x^2 y'' + 3 \sin(x)y' - 2y = 0$

7. $x^2 y'' + \frac{1}{2}(x + \sin x)y' + y = 0$

9. $x^2(1 - x)y'' - (1 + x)y' + 2xy = 0$

11. $(4 - x^2)y'' + 2xy' + 3y = 0$

For 13 and 15, show that $x = 0$ is a regular singular point, find the exponents at that singularity, and find the first three nonzero terms in two linearly independent solutions about $x = 0$.

13. $xy'' + y' - y = 0$

15. $x(x - 1)y'' + 6x^2 y' + 3y = 0$