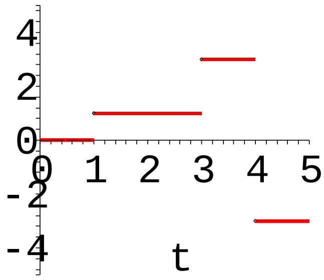
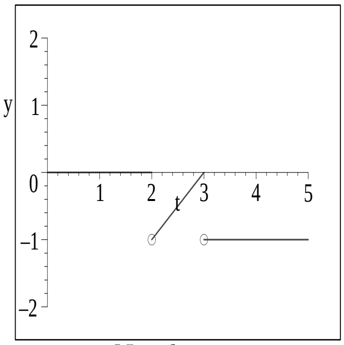
Solutions to Homework Assignment 32

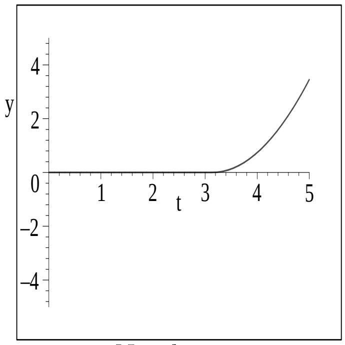
MATH 256-01
Section 6.3, Page 314
Problems: 1-15, 17, 21-27, 29, 31, 32



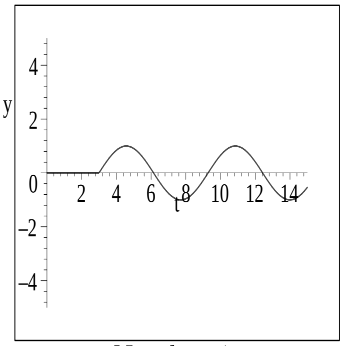
Number 1



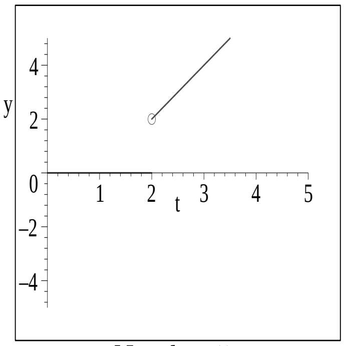
Number 2



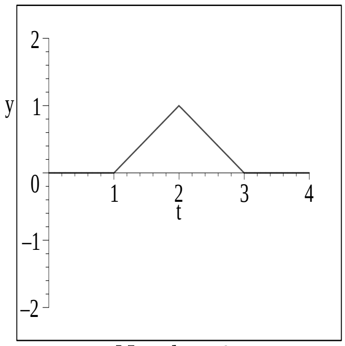
Number 3



Number 4



Number 5



Number 6

7. $f(t) = (t-2)^2 u_2(t)$, so $\mathcal{L}\{f(t)\} = e^{-2t}\mathcal{L}\{t^2\} = \dfrac{2e^{-2s}}{s^3}$.

8. $f(t) = [(t-1)^2+1]u_1(t)$, so $\mathcal{L}\{f(t)\} = e^{-s}\mathcal{L}\{t^2+1\} = e^{-s}\left(\dfrac{2}{s^3}+\dfrac{1}{s}\right)$.

9. $f(t) = (t-\pi)(u_\pi(t) - u_{2\pi}(t)) = (t-\pi)u_\pi(t) - (t-2\pi)u_{2\pi}(t) - \pi u_{2\pi}(t)$, so $\mathcal{L}\{f(t)\} = e^{-\pi s}\mathcal{L}\{t\} - e^{-2\pi s}\mathcal{L}\{t\} - \pi\mathcal{L}\{u_{2\pi}(t)\} = \dfrac{e^{-\pi s}}{s^2} - \dfrac{e^{-2\pi s}}{s^2} - \pi\dfrac{e^{-2\pi s}}{s}$.

10. $\mathcal{L}\{f(t)\} = \dfrac{e^{-s}}{s} + 2\cdot\dfrac{e^{-3s}}{s} - 6\cdot\dfrac{e^{-4s}}{s}$.

11. $f(t) = (t-2)u_2(t) - u_2(t) - (t-3)u_3(t) - u_3(t)$, so $\mathcal{L}\{f(t)\} = e^{-2s}\mathcal{L}\{t\} - \dfrac{e^{-2s}}{s} - e^{-3s}\mathcal{L}\{t\} - \dfrac{e^{-3s}}{s} = \dfrac{e^{-2s}}{s^2} - \dfrac{e^{-2s}}{s} - \dfrac{e^{-3s}}{s^2} - \dfrac{e^{-3s}}{s}$.

12. $\mathcal{L}\{f(t)\} = \frac{1}{s^2} - e^{-s}\mathcal{L}\{t\} = \frac{1}{s^2} - \frac{e^{-s}}{s^2}$.

13. $\mathcal{L}^{-1}\{F(s)\} = t^3e^{2t}$ from Number 11 in the table.

14. Consider just $\frac{1}{s^2+s-2} = \frac{1}{(s+2)(s-1)} = \frac{-1/3}{s+2} + \frac{1/3}{s-1}$. We have $-\frac{1}{3}\frac{e^{-2s}}{s+2} + \frac{1}{3}\frac{e^{-2s}}{s-1}$, so $\mathcal{L}^{-1}\{F(s)\} = -\frac{1}{3}u_2(t)f_1(t-2) + \frac{1}{3}u_2(t)f_2(t-2)$, where $\mathcal{L}\{f_1(t)\} = \frac{1}{s+2}$ and $\mathcal{L}\{f_2(t)\} = \frac{1}{s-1}$. Thus, $f_1(t) = e^{-2t}$ and $f_2(t) = e^t$, so we have $\mathcal{L}^{-1}\{F(s)\} = -\frac{1}{3}u_2(t)e^{-2(t-2)} + \frac{1}{3}u_2(t)e^{t-2}$. (Use Numbers 13 and 2 in the table.)

15. $F(s) = 2\frac{(s-1)e^{-2s}}{(s-1)^2+1^2}$. Number 13 implies that this is the transform of $2u_2(t)f(t-2)$, where $\mathcal{L}\{f(t)\} = \frac{s-1}{(s-1)^2+1^2}$. Therefore, $f(t) = e^t\cos t$ (using Number 10). Thus $\mathcal{L}^{-1}\{F(s)\} = 2u_2(t)e^{t-2}\cos(t-2)$.

16. We have $\mathcal{L}^{-1}\{F(s)\} = u_2(t)f(t-2)$, where $\mathcal{L}\{f(t)\} = \frac{2}{s^2-2^2}$. Thus $f(t) = \sinh 2t$, so $\mathcal{L}^{-1}\{F(s)\} = u_2(t)\sinh 2(t-2)$. (Use Number 7 in the table.) (This one's a bonus!)

17. $F(s) = \frac{e^{-s}(s-2)}{(s-2)^2-1^2}$. Therefore, $\mathcal{L}^{-1}\{F(s)\} = u_2(t)f(t-2)$, where $\mathcal{L}\{f(t)\} = \frac{s-2}{(s-2)^2-1^2}$. Thus, $f(t) = e^{2t}\cosh t$, so $\mathcal{L}^{-1}\{F(s)\} = u_2(t)e^{2(t-2)}\cosh(t-2)$.

21. $F(s) = \frac{2s+1}{(2s+1)^2+2^2} = G(2s+1)$, where $G(s) = \frac{s}{s^2+2^2}$. Part (c) of Number 19 gives $\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{G(2s+1)\} = \frac{1}{2}e^{-1\cdot t/2}g(t/2)$, where $g(t) = \mathcal{L}^{-1}\{G(s)\} = \cos 2t$. Therefore, $\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2}e^{-t/2}\cos t$.

22. $F(s) = \frac{1}{3}\cdot\frac{1}{3s^2-4s+1} = \frac{1}{3}\frac{1}{(3s-1)(s-1)} = \frac{1}{3}\left(\frac{-3/2}{3s-1} + \frac{1/2}{s-1}\right)$. $\mathcal{L}^{-1}\{1\}3s-1 = \frac{1}{3}e^{-(-1)\cdot t/3}f(t/3)$, where $f(t) = \mathcal{L}^{-1}\{1/s\} = 1$. Thus $\mathcal{L}^{-1}\{F(s)\} = \frac{-1}{2}\cdot\frac{1}{3}e^{t/3} + \frac{1}{6}e^t = \frac{1}{6}(e^t - e^{t/3})$.

23. $\mathcal{L}^{-1}\{F(s)\} = e^2u_4(t)f(t-4)$, where $\mathcal{L}\{f(t)\} = \frac{1}{2s-1}$. Using part (c) of 19 again, we find that $f(t) = \frac{1}{2}e^{-(-1)t/2}\cdot 1 = \frac{1}{2}e^{t/2}$. Thus $\mathcal{L}^{-1}\{F(s)\} = \frac{e^2}{2}u_4(t)e^{(t-4)/2} = \frac{1}{2}u_4(t)e^{t/2}$.

24. $\mathcal{L}\{f(t)\} = \mathcal{L}\{1-u_1(t)\} = \frac{1}{s} - \frac{e^{-s}}{s}$.

25. $\mathcal{L}\{f(t)\} = \mathcal{L}\{1-u_1(t)+u_2(t)-u_3(t)\} = \frac{1}{s}(1-e^{-s}+e^{-2s}-e^{-3s})$.

26. $\mathcal{L}\{f(t)\} = \frac{1}{s}(1-e^{-s}+e^{-2s}-e^{-3s}+\ldots+e^{-2ns}-e^{-(2n+1)s}) = \frac{1}{s}\frac{1-e^{-(2n+2)s}}{1+e^{-s}}$.

27. $\mathcal{L}\{f(t)\} = \lim_{n\to\infty}(\text{Answer to Number 26}) = \frac{1}{s}\frac{1}{1+e^{-s}}$.

29.

$$\begin{aligned}
\mathcal{L}\{f(t)\} &= \frac{\displaystyle\int_0^1 1\cdot e^{-st}dt + \int_1^2 0e^{-st}dt}{1-e^{-2s}} \\
&= \frac{\frac{-1}{s}e^{-st}\Big|_0^1}{1-e^{-2s}} \\
&= \frac{\frac{1}{s}(1-e^{-s})}{1-e^{-2s}} \\
&= \frac{1}{s}(1-e^{-s})(1-e^{-s})(1+e^{-s}) \\
&= \frac{1/s}{1+e^{-s}}.
\end{aligned}$$

31.

$$\begin{aligned}
\mathcal{L}\{f(t)\} &= \frac{\displaystyle\int_0^1 te^{-st}dt}{1-e^{-s}} \\
&= \frac{-\frac{1}{s}\left(te^{-st}+\frac{1}{s}e^{-st}\right)\Big|_0^1}{1-e^{-s}} \\
&= \frac{-\frac{1}{s}\left(e^{-s}+\frac{1}{s}e^{-s}-\frac{1}{s}\right)}{1-e^{-s}} \\
&= \frac{1}{s^2}\frac{1-e^{-s}-se^{-s}}{1-e^{-s}}.
\end{aligned}$$

32.

$$\begin{aligned}
\mathcal{L}\{f(t)\} &= \frac{\displaystyle\int_0^\pi \sin te^{-st}dt}{1-e^{-\pi s}} \\
&= \frac{e^{-\pi s}+1}{(s^2+1)(1-e^{-\pi s})}.
\end{aligned}$$

(I used MAPLE for that last integration.)