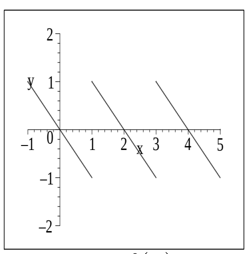
Solutions to Homework Assignment 35

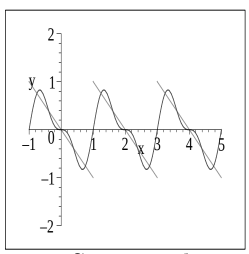
MATH 256-01
Section 10.2, Page 555
Problems: 1-8, 12, 13, 15, 18, 19, 20

1. This is periodic. To find the fundamental period, we need to solve $5x = 2\pi$ for x. We get $x = \dfrac{2\pi}{5}$, and this is the fundamental period.
2. This is periodic with fundamental period $\dfrac{2\pi}{2\pi} = 1$.
3. This is not periodic.
4. This is periodic with fundamental period $\dfrac{2\pi}{\pi/L} = 2L$.
5. This is periodic with fundamental period $\dfrac{\pi}{\pi} = 1$. (The fundamental period of $\tan x$ is π, not 2π.)
6. This is not periodic.
7. This is periodic; its fundamental period is 2. (Consider its graph.)
8. This is periodic with period 4; again, consider its graph.

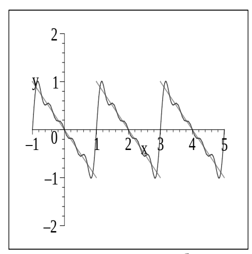
12. I will let you do this. You may thank me later.
13. The graph is below.For (b): Since this is an odd function, $a_n = 0$ for all n. We must determine $b_n = \dfrac{1}{L}\displaystyle\int_{-L}^{L}(-x)\sin\frac{n\pi x}{L}dx$, with $L = 1$. A quick integration by MAPLE gives $b_n = \dfrac{2(-1)^n 2}{n\pi}$. (Recall that $\cos n\pi = (-1)^n$ for n an integer.) Thus, the series is $f(x) = \displaystyle\sum_{m=1}^{\infty}\frac{2(-1)^n}{n\pi}\sin n\pi x$.



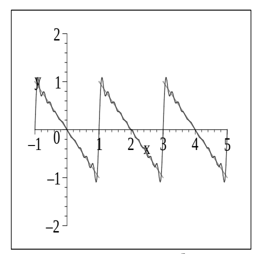
$y = f(x)$



Fourier Series with $n = 2$



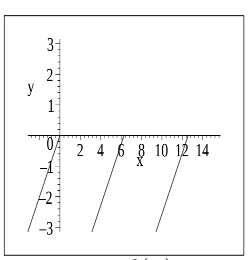
Fourier series with $n = 5$



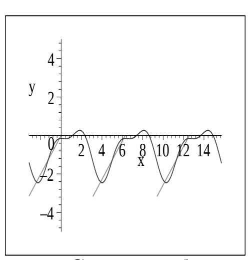
Fourier series with $n = 10$

15. This function is neither odd nor even, so we need to calculate a_n and b_n. I'll ask MAPLE! We have $L = \pi$. Thus $a_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{0} x\cos nx dx = \frac{(-1)^{n+1}+1}{\pi n^2}$ and $b_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{0} x\sin nx dx = \frac{(-1)^{n+1}}{n}$. Also, $a_0 = -\pi/2$. Now

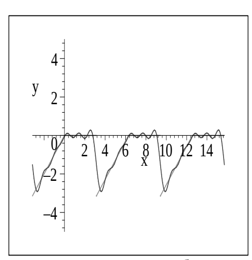
$$f(x) = -\frac{\pi}{4} + \sum_{m=1}^{\infty}\left(\frac{(-1)^{m+1}+1}{\pi m^2}\cos mx + \frac{(-1)^{m+1}}{m}\sin mx\right).$$



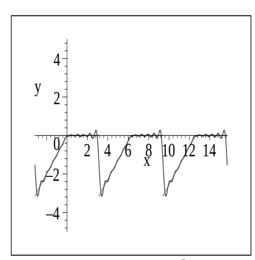
$y = f(x)$



Fourier Series with $n = 2$



Fourier series with $n = 5$

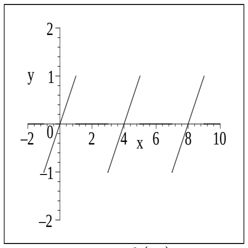


Fourier series with $n = 10$

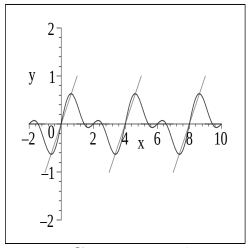
18. This function has odd symmetry, so we only need to compute the b_n. We have $b_n = \dfrac{1}{2}\displaystyle\int_{-1}^{1} x \sin\frac{n\pi x}{4} =$ $\dfrac{2}{\pi^2 n^2}(2\sin(\pi n/2) - \pi n \cos(\pi n/2)$. The series is therefore

$$f(x) = \sum_{m=1}^{\infty} \frac{2}{\pi^2 m^2}(2\sin(\pi m/2) - \pi m \cos(\pi m/2) \sin\frac{m\pi x}{4}.$$

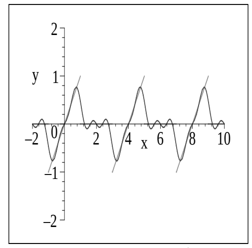
The coefficients can be simplifed since the sine terms are zero for even n and the cosine terms are zero for odd n, but I will let MAPLE do the work.



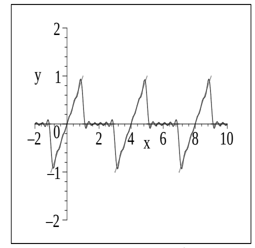
$y = f(x)$



Fourier Series with $n = 2$



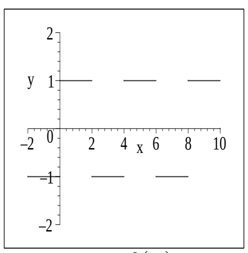
Fourier series with $n = 5$



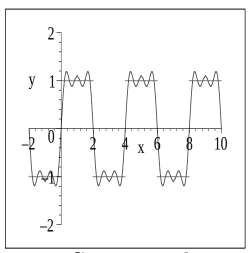
Fourier series with $n = 10$

19. f is odd, so we need only consider the b_n. We have $b_m = \dfrac{1}{2}\displaystyle\int_{-2}^{2} f(x)\sin\frac{m\pi x}{2} = \frac{2((-1)^{m+1}+1)}{m\pi}$. The series is therefore

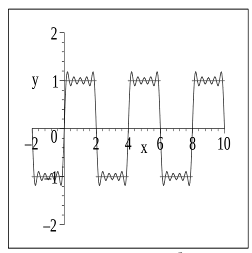
$$f(x) = \sum_{m=1}^{\infty} \frac{2((-1)^{m+1}+1)}{m\pi} \sin\frac{m\pi x}{2}.$$



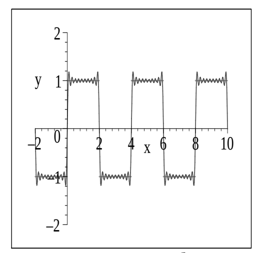
$y = f(x)$



Fourier Series with $n = 5$



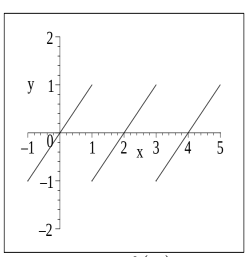
Fourier series with $n = 10$



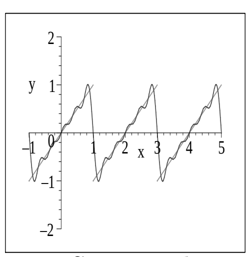
Fourier series with $n = 20$

20. Again, f is odd. We have $b_m = \displaystyle\int_{-1}^{1} x \sin m\pi x = \frac{2(-1)^{m+1}}{m\pi}$, giving

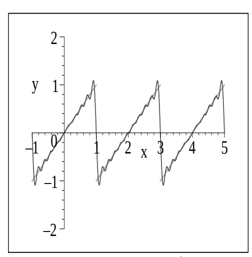
$$y(x) = \sum_{m=1}^{\infty} \frac{2(-1)^{m+1}}{m\pi} \sin m\pi x.$$



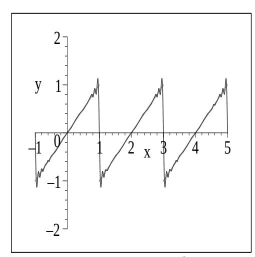
$y = f(x)$



Fourier Series with $n = 5$



Fourier series with $n = 10$



Fourier series with $n = 20$