Solutions to Homework Assignment 4

MATH 345-01

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1. (a) $\dfrac{-2}{1+\sqrt{3}i} = \dfrac{-2+2\sqrt{3}i}{4} = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i = \cos(2\pi/3) + i\sin(2\pi/3)$. Thus, $\text{Arg } z = 2\pi/3$.

   (b) $(\sqrt{3}-i)^6 = (2e^{-i\pi/6})^6 = 64e^{-i\pi}$. Thus $\text{Arg } z = \pi$. (Remember that $-\pi$ is not allowed as a principle argument.)

2. (a) $|e^{i\theta}| = |\cos\theta + i\sin\theta| = \cos^2\theta + \sin^2\theta = 1$.

   (b) $\overline{e^{i\theta}} = \overline{\cos\theta + i\sin\theta} = \cos\theta - i\sin\theta = e^{-i\theta}$.

6. (a) $i(1-\sqrt{3}i)(\sqrt{3}+i) = e^{i\pi/2}\cdot 2e^{-i\pi/3}\cdot 2e^{i\pi/6} = 4e^{\pi/3} = 2(1+\sqrt{3}i)$.

   (b) $\dfrac{5i}{2+i} = 5e^{i\pi/2}\dfrac{1}{\sqrt{5}}e^{-i\arctan(1/2)} = \sqrt{5}e^{i(\pi/2+\arctan(-1/2))} = \sqrt{5}e^{i\arctan(2)} = 1+2i$. I've used the fact that adding $\pi/2$ to the argument rotates the vector by 90 degrees, which means the end result should be perpendicular to the original. I've also used the fact that the slopes of perpendicular lines are negative reciprocals of each other.