Solutions to Homework Assignment 5

MATH 345-01

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1. (a) $2i = 2e^{i\pi/2}$, so the square roots are $\sqrt{2}e^{i\pi/4} = 1 + i$ and $\sqrt{2}e^{i(\pi/4+2\pi/2)} = -1 - i$.

   (b) $1 - \sqrt{3}i = 2e^{-i\pi/3}$, so the square roots are $\sqrt{2}e^{-i\pi/6} = \dfrac{\sqrt{3} - i}{\sqrt{2}}$ and $\sqrt{2}e^{-i\pi/6+i\pi} = \dfrac{-\sqrt{3} + i}{\sqrt{2}}$.

3. (a) $-16 = e^{i\pi}$, so $(-16)^{1/4} = 2e^{i\pi/4+2\pi ik/4} = \sqrt{2}(1 + i), \sqrt{2}(-1 + i), \sqrt{2}(-1 - i), \sqrt{2}(1 - i)$.

   (b) $-8 - 8\sqrt{3}i = 16e^{-2\pi i/3}$, so $(-8 - 8\sqrt{3}i)^{1/4} = 2e^{-2\pi i/12+2\pi ik/4} = \pm(\sqrt{3} - i), \pm(1 + \sqrt{3}i)$.

6. $(-4)^{1/4} = \sqrt{2}e^{i\pi/4+2\pi ik/4} = \pm(1+i), \pm(1-i)$. Thus $z^4+4 = (z-1-i)(z-1+i)(z+1-i)(z+1+i) = (z^2 - 2z + 2)(z^2 + 2z + 2)$.