Solutions to Homework Assignment 7

MATH 345-01

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1, 2a, 3, 5, 8

1. (b) $z \neq 0$ [Note: Answer (b) in the book is actually for part (c).]

   (d) $|z| \neq 1$. (Everything except the unit circle.)

2. (a) $(x+iy)^3 + (x+iy) + 1 = (x^3 - 3xy^2 + x + 1) + i(3x^2y - y^3 + y)$.

3. $\dfrac{(z+\overline{z})^2}{4} - \dfrac{(z-\overline{z})^2}{-4} - \dfrac{(z-\overline{z})}{i} + i\left(z + \overline{z} - \dfrac{z^2 - \overline{z}^2}{2i}\right) = \overline{z}^2 + 2iz.$

5. Recall that $f(z) = z^2$ maps hyperbolas $x^2 - y^2 = c > 0$ to vertical lines $u = c$ and hyperbolas $2xy = c > 0$ to horizontal lines $v = c$. Thus, we want the region bounded by the hyperbolas $x^2 - y^2 = 1, x^2 - y^2 = 2, 2xy = 1$, and $2xy = 2$.

8. The given set is one eighth of the unit disk. Squaring maps it onto one quarter (the first quadrant), cubing maps it onto $3/8$, and z^4 maps it onto the upper half-plane along with the real axis.