Solutions to Homework Assignment 8

MATH 345-01

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1. (a) Let $\epsilon > 0$ be given, and let $\delta = \epsilon$. Then $0 < |z - z_0| < \delta \implies |\Re(z) - \Re(z_0)| = |\Re(z - z_0)| \leq |z - z_0| < \delta = \epsilon$, as desired. Thus $\lim_{z \to z_0} \Re z = z_0$.

   (b) Let $\epsilon > 0$ be given, and let $\delta = \epsilon$. Then $0 < |z - z_0| < \delta \implies |\overline{z} - \overline{z_0}| = |\overline{z - z_0}| = |z - z_0| < \delta = \epsilon$, as desired. Thus $\lim_{z \to z_0} \overline{z} = \overline{z_0}$.

   (c) Let $\epsilon > 0$ be given, and let $\delta = \epsilon$. Then $0 < |z - 0| < \delta \implies \left| \frac{\overline{z}^2}{z} - 0 \right| = \frac{|z|^2}{|z|} = |z| < \ delta = \epsilon$, as desired.

3. You can do these.

10. (a) $\lim_{z \to \infty} \frac{4z^2}{(z-1)^2} = \lim_{z \to 0} \frac{4/z^2}{(1/z - 1)^2} = \lim_{z \to 0} \frac{4}{(z-1)^2} = 4.$

    (b) $\lim_{z \to 1} \frac{1}{(z-1)^3} = \infty \iff \lim_{z \to 1} \frac{(z-1)^3}{1} = 0$, which is true.

    (c) $\lim_{z \to \infty} \frac{z^2 + 1}{z - 1} = \infty \iff \lim_{z \to 0} \frac{1/z - 1}{1/z^2 + 1} = 0 \iff \lim_{z \to 0} \frac{z - z^2}{1 + z^2} = 0$, which is true.

11. T is what's called a **linear fractional transformation** or a **fractional linear transformation.** These have lots of nice properties.

    (a) If $c = 0$, then $\lim_{z \to 0} \frac{d}{a/z + b} = \lim_{z \to 0} \frac{dz}{a + bz} = 0$, so $\lim_{z \to \infty} T(z) = \infty$. (Note that if $c = 0$, then $a \neq 0$ by the assumption $ad - bc \neq 0$.)

    (b) $\lim_{z \to 0} T(1/z) = \lim_{z \to 0} \frac{a/z + b}{c/z + d} = \lim_{z \to 0} \frac{a + bz}{c + dz} = \frac{a}{c}$ and $\lim_{z \to -d/c} \frac{cz + d}{az + b} = 0$, as desired.