

Solutions to Homework Assignment 11

MATH 345-01

Section 26, Page 73

1,2,4,5

1. (a) $u_x = 3 = v_y, u_y = 1 = -v_x$. Since these are continuous everywhere, f is entire.
(b) $u_x = \cos x \cosh y = v_y, u_y = \sin x \sinh y = -v_x$. Since these are continuous everywhere, f is entire.
(c) Part (c) is similar.
(d) $f(z) = \frac{z^2 - 2}{e^z}$. This is entire since e^z is nonzero in \mathbb{C} . I know that's not the theorem they're talking about, but forget that!
2. (a) $u_x = y \neq 1 = v_y$ unless $y = 1$. Thus it cannot be analytic in any neighborhood.
(b) $u_x = 2y \neq -2y = v_y$ unless $y = 0$. Thus it cannot be analytic in any neighborhood.
(c) $u_x = -e^y \sin x, v_y = e^y \sin x$. These are only equal for $\sin x = 0$, which occurs at discrete points (not neighborhoods).
4. Each of these is rational, so each is analytic on its domain. The singular points are given in the text; they come from zeroes of the denominators.
5. Notice that if $x > 1$, then $2z - 2 + i = 2(x - 1) + i(2y + 1)$ has positive real part. Thus we may take its argument between $-\pi/2$ and $\pi/2$, and its outputs are in the domain of g . Therefore $G(z)$ is analytic, and $G'(z) = g'(2z - 2 + i)(2) = \frac{1}{g(2z - 2 + i)}$.
(The thing we really need to watch out for is that $2z - 2 + i$ includes *no part* of the non-positive real axis.)