Solutions to Homework Assignment 14

MATH 345-01

Section 30, Page 89

1,2,8,10

1. (a) $e^{2\pm 3\pi i} = e^2(\cos 3\pi \pm i \sin 3\pi) = -e^2$.

   (b) $e^{(2+\pi i)/4} = e^{1/2}(\cos \pi/4 + i \sin \pi/4) = \dfrac{\sqrt{2}\sqrt{e}}{2}(1+i) = \sqrt{e}2(1+i)$.

   (c) $e^{z+\pi i} = e^z e^{\pi i} = -e^z$.

2. $2, z^2, 3, z, e^z$, and e^{-z} are all entire, so any sum of products of them is also entire by theorems in section 19.

8. (a) $z = \log -2 = \ln|-2| + i\arg(-2) + \ln 2 + i(\pi + 2k\pi)$.

   (b) $z = \log(1+\sqrt{3}i) = \ln(2) + i\arg(1+\sqrt{3}i) = \ln 2 + i(\pi/3 + 2k\pi)$.

   (c) $2z - 1 = \log 1 = i(2k\pi)$, so $z = k\pi i + \dfrac{1}{2}$.

10. (a) Suppose that $e^z = e^x(\cos x + i \sin y)$ is real. Then $\sin y = 0$, so $y = k\pi$ for some integer k. Since $y = \Im z$, we have $\Im z = k\pi$.

    (b) Suppose that $e^z = e^x(\cos x + i \sin y)$ is pure imaginary. Then $\cos y = 0$, so $y = (2k+1)\pi/2$ for some integer k. Since $y = \Im z$, we have $\Im z = (2k+1)\pi/2$.