Solutions to Homework Assignment 17

MATH 345-01

Section 36, Page 103

1,2,3,5,7,8ac,9

1. (a) $(1+i)^i = e^{i\log(1+i)} = e^{i\ln\sqrt{2}-\pi/4+2n\pi} = e^{-\pi/4+2n\pi}e^{i(\ln 2)/2}$.

   (b) $\dfrac{1}{i^{2i}} = \left(\dfrac{1}{i^i}\right)^2 = e^{-2i\log(i)} = e^{-2i(\ln(1)+i(2\pi n+\pi/2)} = e^{4\pi n+\pi}$.

2. (a) $(-i)^i = e^{i\log(-i)} = e^{i(\ln(1)-i(2\pi n+\pi/2)}$. The principle value has $n=0$, so we get $e^{\pi/2}$.

   (b)

$$\begin{aligned}\left(\frac{e}{2}(-1-\sqrt{3}i)\right)^{3\pi i} &= \exp\left(3\pi i\left(\ln e - i2\pi/3\right)\right)\\ &= \exp\left(3\pi i - 2\pi^2\right)\\ &= -e^{2\pi^2}.\end{aligned}$$

   (c) $(1-i)^{4i} = e^{4i(\ln\sqrt{2}-\pi i/4)} = e^{\pi}e^{2i\ln 2}$.

3. $(-1+\sqrt{3}i)^{3/2} = e^{(3/2)(\ln 2+2\pi i/3} = e^{3\ln 2/2}e^{\pi i} = -e^{(\ln 8)/2} = -8^{1/2} = \pm 2\sqrt{2}$.

5. For the principal root, we have $z_0^{1/n} = e^{(1/n)\text{Log } z_0} = e^{(1/n)(\ln|z_0|+iArgz_0)} = \sqrt[n]{|z_0|}e^{i\Theta/n}$. From Section 8, we have $z_0^{1/n} = \sqrt[n]{|z_0|}e^{i\Theta/n+2k\pi/n}$, and taking the principal value of this gives the desired expression.

7. $|i^c| = |e^{c\log i}| = |e^{(a+bi)(i\pi/2+i2k\pi)}| = |e^{-b\pi/2-2kb\pi}e^{a(i\pi/2+i2k\pi)} = e^{-b\pi/2-2kb\pi}$. This varies with k unless $b=0$, so we require that c be real.

8. (a) $z^{c_1}z^{c_2} = e^{c_1\text{Log } z}e^{c_2\text{Log } z} = e^{(c_1+c_2)\text{Log } z} = z^{c_1+c_2}$.

   (c) We have $(z^c)^n = (e^{c\text{Log } z})^n = e^{nc\text{Log } z}$ from earlier work ($(e^z)^n = e^{nz}$). Thus we get z^{nc}, as desired.

9. $\dfrac{d}{dz}c^{f(z)} = \dfrac{d}{dz}e^{f(z)\log c} = e^{f(z)\log c}f'(z)\log c = c^{f(z)}f'(z)\log c$.