Solutions to Homework Assignment 20

MATH 345-01

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1. (a) $\arctan(2i) = \dfrac{i}{2}\log\dfrac{i+2i}{i-2i} = \dfrac{i}{2}\log(-3) = \dfrac{i}{2}(\ln 3 + i(2n+1)\pi) = -\dfrac{(2n+1)\pi}{2} + \dfrac{i\ln 3}{2}$.

   (b) $\arctan(1+i) = \dfrac{i}{2}\log\dfrac{2i+1}{-1} = \dfrac{i}{2}(\ln\sqrt{5} + i(\arctan(-2) + 2k\pi)) = \dfrac{i}{4}\ln 5 + \arctan 2 - 2k\pi$.

   (c) (c) and (d) are similar.

2. $\sin z = 2 \implies z = \arcsin 2 = -i\log(2i + (-3)^{1/2}) = -i\log(2i \pm \sqrt{3}i) = -i(\ln(2 \pm \sqrt{3}) + i\pi/2 + 2k\pi i)$. Notice that Arg $(2i + \sqrt{3}i) =$ Arg $(2i - \sqrt{3}i) = \pi/2$ since $2 - \sqrt{3} > 0$. Thus $z = -i\ln(2 \pm \sqrt{3}) + \pi(2k + 1/2) = \pm i\ln(2 + \sqrt{3}) + \pi(2k + 1/2)$ (since $2 - \sqrt{3} = \dfrac{1}{2+\sqrt{3}}$).

3. $\cos z = \sqrt{2} \implies z = \arccos(\sqrt{2}) = -i\log(\sqrt{2} + i(-1)^{1/2}) = -i\log(\sqrt{2} \pm 1) = -i(\ln(\sqrt{2} \pm 1) + 2k\pi i) = -i\ln(\sqrt{2} \pm 1) + 2k\pi = \pm i\ln(\sqrt{2} + 1) + 2k\pi$.