Solutions to Homework Assignment 22

MATH 345-01

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1. (a) $\displaystyle\int_{-b}^{-a} w(-t)dt = \int_{-b}^{-a} u(-t)dt + i\int_{-b}^{-a} v(-t)dt$. Let $\tau = -t$. Then $d\tau = -dt$, and we have $\displaystyle -\int_{b}^{a} u(\tau)d\tau - i\int_{b}^{a} v(\tau)d\tau = \int_{a}^{b} u(\tau)d\tau + i\int_{a}^{b} v(\tau)d\tau$, as desired.

   (b) Let $t = \phi(\tau)$. Then $dt = \phi'(\tau)d\tau$, so we get

$$\begin{aligned}\int_a^b w(t)dt &= \int_a^b u(t)dt + i\int_a^b v(t)dt \\ &= \int_\alpha^\beta u(\phi(\tau))\phi'(\tau)d\tau + i\int_\alpha^\beta v(\phi(\tau))\phi'(\tau)d\tau \\ &= \int_\alpha^\beta w(\phi(\tau))\phi'(\tau)d\tau,\end{aligned}$$

   as desired.