Solutions to Homework Assignment 24

MATH 345

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1. (a) $\left|\int_C \frac{z+4}{z^3-1}dz\right| \le \int_C \left|\frac{z+4}{z^3-1}dz\right| \le \int_0^{\pi/2} \frac{2(|z|+4)}{||z^3|-1|}dt = \frac{12}{7}\frac{\pi}{2} = \frac{6\pi}{7}$.

   (b) $\left|\int_C \frac{dz}{z^2-1}\right| \le \int_C \left|\frac{dz}{z^2-1}\right| \le \int_0^{\pi/2} \frac{2dt}{||z^2|-1|} = \frac{1}{3}(\pi)$ (since the length of the arc is π).

3. $\left|\int_C e^z - \overline{z}\right| \le \int_C |e^z - \overline{z}| \le \int_a^b (|e^z| + |\overline{z}|)|z'(t)|dt$. (I've written the last part using a generic a and b just to simplify things.) $|e^z| = |e^x|$, so on the given contour, $|e^z| \le e^0 = 1$. $|\overline{z}| = |z|$, which is largest when z is farthest from the origin; this happens at $z = -4$. Thus the modulus of our integral is less than or equal to $(1+4)L = 5L$, where L is the perimeter of the triangle. The perimeter is $3 + 4 + 5 = 12$, so the modulus is bounded by 6, as desired.