Solutions to Homework Assignment 26

MATH 345

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1. (a) The only singularity is at $z = 3$, so the integrand is analytic on and inside C.

   (b) The integrand is entire.

   (c) The singularities of f are at $-1 \pm i$, both of which are outside the unit circle.

   (d) $\operatorname{sech} z = \dfrac{1}{\cosh z}$, so the singularities are at the zeros of $\cosh z$. These are at $\dfrac{\pi}{2}i + n\pi i$, all of which are outside the unit circle.

   (e) The singularities of $\tan z$ are at $\dfrac{\pi}{2} + n\pi$, all of which are outside the unit circle.

   (f) $\operatorname{Log}(z + 2)$ has a branch point at $z = -2$ and the branch cut is to the left of that. Thus $\operatorname{Log}(z + 2)$ is analytic on and inside C.

2. (a) The singularities are at $z = \pm\dfrac{1}{3}i$, both of which lie inside the square. Thus f is analytic between the two curves.

   (b) The singularities are at $2k\pi$, none of which occur between the two curves.

   (c) The singularities are at $e^z = 1$, which occurs for $z = 0, 2k\pi i$. Again, none of these lie between the two curves.