

# Solutions to Homework Assignment 27

MATH 345

Section 57, Page 170

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1. (a) Since  $\pi i/2$  lies inside the square, this is  $2\pi i e^{-\pi i/2} = 2\pi$ .
  - (b) The only singularity inside the square is  $z = 0$ , so we get  $2\pi i \frac{\cos 0}{0^2 + 8} = \frac{\pi i}{4}$ .
  - (c) Since  $z = -1/2$  is inside the square, we get  $\int_C \frac{z/2}{z + 1/2} dz = 2\pi i(-1/2)/2 = -\pi i/2$ .
  - (d)  $(\cosh z)''' = \sinh z$ , and  $\sinh 0 = 0$ , so the integral is 0.
  - (e)  $(\tan(z/2))' = \frac{1}{2} \sec^2(z/2)$ , so the integral is  $2\pi i \frac{1}{2} \sec^2(x_0/2) = i\pi \sec^2(x_0/2)$ .
3. If  $|w| > 3$ , then  $g(w) = 0$ . If  $|w| < 2$ , then  $g(w) = 2\pi i(2w^2 - w - 2)$ . For  $w = 2$ , this is  $8\pi i$ .